Managed Competition as an Incentive Mechanism
in Supply Relations♠

by

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Abstract

This paper theoretically examines a procurement form in supply relations, called 'Managed Competition', where a principal (buyer) uses dynamic competition between two agents (suppliers) to maximize its payoff. The principal commits to an incentive scheme at the beginning, and assigns the production share in the final period, according to the interim rank of the level of capital accumulation. By that, an asymmetric equilibrium favorable to the winner is generated in the subsequent stage, which in turn creates a tournament effect through a discrete prize in the first period, in addition to a marginal strategic effect. We characterize the optimal solution to the overall problem (a Kuhn-Tucker problem with static and dynamic incentive constraints) faced by the principal, and investigate some conditions, under which the principal wants to induce ex post competition among agents actively, under which she doesn't induce such competition and restrains procurement from the loser, and under which she chooses not to intervene in the form of 'allotment', even with observability of the progress along the way among agents. These solutions imply that the principal endogenously chooses the mode of competition, depending upon the exogenous conditions. This kind of intervention mechanism not only promotes accumulation of relation specific skills, but induces it in a less costly manner. It also explains an aspect of efficiency in managed supplier organization in the Japanese automobile industry (as a typical example, *two supply policy* and *design in* of Toyota), essential aspects of which, in the 1980's and early 1990's, the Big Three in the United States automobile industry (GM, Ford, Chrysler) have tried to learn and adopt. Last, as a comparative statics, we investigate how (1) the bargaining powers of suppliers and (2) the degree of technology transfer after the interim ranking established may affect the mode of managed competition, including dynamic investment incentives.

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1. Introduction

As a source of international competitiveness of the Japanese automobile industry seen in the 1980's, trading relationships for intermediate goods between assemblers and parts suppliers, that is, *supply relations* have attracted international attention. Thereafter, in the early 1990's, as was greatly impressive, the Big Three in the United States automobile industry (Chrysler, Ford, GM) tried to learn and adopt essential aspects of Japanese supply relations. The supply relations in the Japanese automobile industry are such that on the one hand, there exists a tight, longstanding and stable relationship between an auto maker and parts suppliers, and on the other hand, under a commitment (promise) by the assembler as a buyer of intermediate goods, a few of the parts suppliers are managed by a *Visible Hand*, resulting in a form of competition among them, and compete fiercely with each other, each seeking a position as the parts supply source for a car model.[1]

As a typical example of the managed supplier organization, Milgrom and Roberts (1992) gives one, that of Toyota which has a *two supply policy*. Except for products produced exclusively in- house, at least two suppliers always exist for each category of component, and those suppliers are comparatively evaluated, according to how well they have performed. In some categories of parts, Toyota Company selects suppliers before the parts specification are made final. If supplier A makes head lamps (brakes, or tires) for model X, then a different supplier B will make the head lamps (brakes or tires) for some other model Y. Having just one supplier for the head lamps (brakes, or tires) of a single model allows the supplier to take full advantage of economies of scale in making the part. Guaranteeing the supplier that it will continue to make the part through the life of the model also helps to protect to a degree any specific investment the supplier makes. However, having a different supplier for the other model allows Toyota to use *comparative performance evaluation* on cost and quality of parts, and then the top performers are rewarded in the form of contract offers with additional orders to make, for example, head lamps and tail lamps on the next car model, or lead lamps on two different models (provided that the two supplier policy is not violated). If the supplier demonstrates technical proficiency, the reward may be an *upgrading* that qualifies the supplier to make more complex (high ranked) parts. This 'two supplier' policy would work so as to generate competitive pressure toward low price and high quality parts between suppliers.

Itami (1988), Asanuma (1989) and Clark and Fujimoto (1991) point out this practice in their field studies. In

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[1] Helper (1991, 1994) applies the concept by Hirchman (1970) 'Voice and Exit' mechanisms, to the transactional relationships between automobile companies and parts suppliers in Japan and U.S, and classifies them into 'Voice' mechanism-type (Japan) and 'Exit' mechanism-type (U.S). She points out the phenomenon that U.S transactional relationship between firms in the automobile industry shifts towards the Japanese-type (a 'voice' mechanism), by their 'catch-up' efforts. Holmstrom and Roberts (1998) also point it out, and theoretically discusses the comparison of traditional procurement and subcontracting practices across the U.S, and Japanese automobile industries. Motivation of this paper is close to theirs, though we had already written another paper (Konishi, Okuno-Fujiwara, and Suzuki (1996)).

addition, they report that one of the other main characteristics of Japanese auto-parts transaction is that parts suppliers are not only delegated the production of parts, but also they play an important role in *development* of components, and are actively involved from an early stage of the process of parts development in the automobile companies. Asanuma (1989) and Clark and Fujimoto (1991) classified automobile parts mainly into two categories, from the viewpoint of how the degree of *technological initiative* is assigned between an assembler and a parts supplier in activities pertaining to the development of each part (component). According to Asanuma (1989), there is a dichotomy of purchased parts, based on the function exercised by suppliers. One category is (1) parts manufactured by outside suppliers according to *drawings supplied* made by the core firm (assembler). The other category is (2) parts manufactured by outside suppliers, according to *drawings* made by the respective suppliers and *approved* by the core firm (assembler). Since the drawings in (1) and (2) are called "drawings supplied " and "drawings approved" respectively, Asanuma called parts (1) and (2) DS parts and DA parts, respectively. DS suppliers are providing basically only capabilities for *manufacturing* of the parts transacted, and their technological initiative is lower, while DA suppliers are providing capabilities for *product development* as well, and so the degree of technological initiative in their case is higher. In Japanese supply relations, it is reported that the proportion of DA parts has expanded compared with DS parts and Market parts. By adopting a form of DA parts more often, Japanese automobile companies can select the first-tier suppliers *before* parts specifications are made final, which makes it possible to exploit the suppliers' expertise in *design engineering* in order to design parts, and design/adjust parts with consideration given to real productivity. This would lead to an improved speed of development and quality improvement. Asanuma (1989) further refines the classification of DA parts, according to a ranking based on the degree of technological initiative, which is exerted by suppliers in the development process. We have noted this parameter, (a degree of) technological initiative, and as far as we know, there has not been any theoretical analysis done to assess how the mode of competition between suppliers under managed competition can be affected, depending on ranks of parts.

In Japanese supply relations, there is another characteristic, concerning the form of exchange of information between suppliers and an automobile company. In development activities, called "Design-in", where parts suppliers are actively involved from the initial stage, information on the technology that engineers employed by parts suppliers may hold would be shared to a high degree, both through joint work with those employed by an automobile company for design engineering and through continual improvement activities until final approval of the drawings of DA parts. In development competition between two suppliers, there also occurs an exchange and transfer of technology information among them, through an automobile company's offering the opportunity for transferring and learning such information, after the interim ranking has been established. Hence, this system may encourage suppliers to be innovative, but also promote rapid communication of valuable information among competing suppliers. Further, this information transfer/sharing would involve not only competing suppliers of the same component, but also suppliers of other different components, in other words, the supply relation as a whole. Since it is another important aspect of "Managed Competition" to incorporate such an institution of information

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transfer/sharing,[2] we should investigate theoretically how the degree of information transfer/sharing can affect the mode of "Managed Competition".

Our model of "Managed Competition" reflects in its structure such a practice that automobile companies go on with a lot of development, with several stages overlapping with one another, and they let suppliers become actively involved in the development of parts from an early stage, and they take advantage of suppliers' design capabilities and the peculiarities of their equipment. This model of "Managed Competition" is a theoretical analysis of a business model with some universality, in that the Big Three in the U.S automobile industry had learned and adopted to a degree its essential aspects in the early 1990's. Despite several recent changes in transaction relationships, the system of "Design-in" with suppliers involved at an early stage in development remains unchanged. In fact, automakers increasingly tend to urge competitive suppliers to be involved in development activities as early as possible. Also from this viewpoint, the paper deals with a current topic. Even in terms of recent tendency of *modularization*, where each module is procured *in lot* from one module-maker, the status as a supply source of a component for suppliers trading with a module-maker might not be stable, since there exists a pressure of entry by other competitive innovative suppliers, and so fierce competition would still exist. In sum, in our view it seems likely that, while auto-makers go on with a foreclosure of competitive suppliers more than before, such subsystems in supply relations as "Managed Competition" and "Design-in" will remain as an effective organizational form.

The organization of this paper is as follows. First, in section 2, based on the above motivation, we construct a model of "Managed Competition" in supply relations as a two-stage procurement-contracting model. Then, in section 3.1 through 3.3, we solve the model by backward induction. The overall problem is mathematically a Kuhn=Tucker problem, with several (static and dynamic) incentive constraints and an information management policy. The solution to the problem corresponds to the situation, where the principal *endogenously* chooses the *mode of dynamic competition* among the competitive suppliers, depending on the exogenous parameters. Then, using this model, we investigate how the mode of "Managed Competition" may vary, according to the degree of technological initiative exerted by each supplier and the degree of information transfer/sharing. In section 4, several theoretical extensions are presented. Finally in section 5, we conclude the paper.

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[2] According to Helper (1991, 1994), this close information sharing/transfer is an important aspect of 'voice' mechanism, together with another aspect that parts suppliers 'trust' core firms (assemblers) in longstanding and stable relationships. We don't explicitly incorporate the latter aspect into the analysis, e.g., in the framework of repeated games, but briefly remarks at the section 4.2, 'Incentive constraints (Self-enforcing constraints) for the principal'. Holmstrom and Roberts (1998) also discuss this point as a major problem for the 'two-supplier' policy to work.

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2. The Model

Now, we consider two sets of risk neutral players. Let us call them the principal and the agents. The setting can be considered as a model of transactional relationships a`la Grossman and Hart (1986) and Bolton and Whinston (1993). As an example, we can establish a model of the parts transactional relationships such as in the case of Manufacture-Supplier Relationships in Japan, but the model is applicable to the other contexts such as "second sourcing" in the U.S defense procurement. The former of the two sets of players is the buyer (assembler), and the latter is the set of potential sellers (suppliers), where the sellers (suppliers) supply the parts needed for the production of the final good (e.g. the automobile) to the buyer (assembler). The principal (the buyer or assembler) has an essential resource (e.g., assembling technology, quota of ordered quantities, specific capital etc.) and the suppliers, by acquiring the favorable contracting opportunity, can increase its competitive position and the subsequent payoff. The principal exercises the discretion over the organization structure and under that, the suppliers invest in the accumulation of relation specific skills, including the decision as to whether to participate. These investments are specific for the buyer. We assume that the principal can transact with the two agents over time.[3] (Of course, both parties have <u>the decision option to terminate the relationship along the way</u>.)

Figure 1 around here

As you see from figure 1, in the basic model, there exist <u>two periods</u> before the production and the sales occur. The accumulated capital stock remains unknown [4]until the end of each period.

Assumption 1: Linear Capital Accumulation Technology

The capital stock of agent i at the end of the first period is represented by the following stochastic variables.

(A1) $\qquad \widetilde{K}_{i1} = \overline{K} + h_i + \varepsilon_i \qquad i = 1,2$ $\qquad\qquad\qquad\qquad$ (1)

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[3]Konishi et al (1996) explicitly derives a result that the "optimal" number is two, in a similar model.

[4] This implies, <u>in terms of the information structure</u>, that the agents adopt *open loop (path) strategies* where they commit themselves to a capital accumulation path during the period .We discuss more about this from the viewpoint of open-loop vs. closed loop strategies. See section 4.5.

$\overline{K}$ is the capital stock at the start point, observable among the principal and two agents. h_i is the investment level during the first period. ε_i is the uncertainty factor (the random shock in the first period). The expectation of ε_i is $E(\varepsilon_i) = 0$, and the variance is $E(\varepsilon_i^2) = \sigma^2$. ε_i is independent of *any other* variable, including other ε_j. Thus, the noises are assumed to be identically and independently distributed. Next, the capital stock of agent i at the end of the second period is represented by the following stochastic variables.

(A1') $\qquad\qquad \widetilde{K}_{i2} = K_{i2} + e_i + \varepsilon_i \qquad\quad i = 1,2 \qquad\qquad\qquad\qquad$ (1)'

Here, K_{i2} is the modified capital stock of agent i, after the transfer (spillover) of technology and knowledge incorporated between the end of the first period and the start of the second period. As is known from this equation, the final capital stock $\widetilde{K}_{i2}$ consists of the sum of the start of second period capital K_{i2}, the investment e_i and a noise $\varepsilon_i, i = 1, 2$. Also in the second period, the expectation of ε_i is $E(\varepsilon_i) = 0$, and the variance is $E(\varepsilon_i^2) = \sigma^2$. ε_i is independent of *any other* variable, including other ε_j. Thus, the noises in the first period and the second period are also assumed to be identically and independently distributed. Further, for the simplicity of the analysis, we assume that the noises in both periods have <u>no lasting value</u> such as accumulated capital stock, consisting of the initial capital stock and the investment. In other words, **the noises only affect the ranking decision by the principal**.[5] The principal and the agents know the distribution of ε. They are common knowledge.

Assumption 2: Cost Functions of Investment in Both Periods

(A2) The cost of relation specific investment in the first period is $g(h_i), i = 1,2$,

   where $g'(h_i) > 0,$ and $g''(h_i) > 0$.

(A2') The cost of relation specific investment in the second period is $C(e_i), i = 1,2$ (or W, L),

   where $C'(e_i) > 0$, $C''(e_i) > 0$ and $C'''(e_i) \geq 0$.

We set the third part of (A2'); $C'''(e_i) \geq 0$ in order to make clear the comparative statics results on the

equilibrium outcome of the second stage.

Timing of the Game and Interpretations of the Setting

   The timing of the sequence of events is summarized as follows. First, the principal proposes an initial contract (incentive scheme) intended for both agents. The contents are two-folds. One of them

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[5] As examples, imagine the measurement errors on the principal's side or some factors which make the presented quality of parts look better than what it really is, such as some favorable impression by a casual greeting and/or gift, or good presentation of the quality of parts.

is the division of production volume (the ordered quantity) of the final period among the agents $(\psi, 1-\psi)$, according to the interim rank. The other is the monetary payment (prize) W, depending upon the final rank of the capital accumulation competition. As interpretations of W, we can imagine the following things. The first interpretation is as follows. If a supplier demonstrated a technical proficiency, then the reward might be an *upgrading* that qualified the supplier to make more complicated (high-ranked) parts. Then, we can interpret that W implies the expected continuation value brought about by *upgrading*. The second interpretation is that the (final) top performer in the competition is rewarded in the form of the contract offers with additional orders to make, for example, head lamps and tail lamps on the next car model, or head lamps on two different models. Then, W implies an additional revenue by the increase of orders and jobs.

Repeatedly speaking about the timing of the incentive schemes $(\psi, 1-\psi)$ and W, the former is basically according to the relative performance (rank order) of the first period competition and the latter is according to the final rank based upon the outcome of the second period competition.[6] Remember that the relative position of the first period capital accumulation levels, represented by (1), determines the interim rank, and the relative position of the final capital accumulation levels, represented by (1)', determines the final rank.

   After observing this rule, the agents accept or reject it[7]. The suppliers are identical *ex ante* in the capital level of $\overline{K}$, and they invest in the capital accumulation simultaneously and independently, in each of two periods, given the production allotment scheme and the rule for the monetary payment (prize), which the principal has offered and committed himself/herself to.

   The investments in the capital accumulation by the agents are unverifiable at the court, so, non-contractible. The idea of incorporating the investment over two periods before the production and the sales reflects the situation of continuous investments and updating the quality of product. The interpretation of the second period investment e could be considered as follows. Each supplier, even after selected and assigned a production share (allotment) as a parts supplier of a model of a car, must make a quality improvement investment and/or adjustment investment with the assembler until finalizing the parts specification. The latter investment could be *an operative effort* before production. If a parts supplier showed the better (final) performance, even though he

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were a follower at the start of the second period, he would be given some *reward* for his final victory in the competition, the size of which is just W. This might be considered as a modeling of the unique Japanese transactional form, where the asset of technological know-how and the production skills are continuously generated, improved and updated. At the final stage given the accumulated capital configurations, if both sides desire the trade, then the production is carried out, and the traded goods generate the expected value of (1')

$$E\left(\widetilde{K}_{i2}\right) = K_{i2} + e_i, i = 1,2$$ per unit production. We assume that the unit production (and sales) cost is zero.

Hence, the gross joint surplus of trade is the expectation of the final accumulated quality represented by (1').

The division of surplus is decided by the bargaining between them.

Assumption3: *At the time of sharing production gains, the principal and each agent split the gain from trade among them, according to* <u>*Nash bargaining*</u>. *The bargaining power* $\alpha \in [0,1]$ *is then exogenously given in our model, which reflects a degree of technological initiative on the supplier's side.*

The bargaining power $\alpha \left(0 \leq \alpha \leq 1\right)$ of each agent, though it is exogenously given, has an important implication when interpreting the model. On the interpretation of α, there is an existing literature. Riordan (1991) gives a specific treatment of the manufacturer-supplier relationship, and a particular kind of vertical integration: backward integration. A large downstream firm may acquire claims on small upstream firm (supplier)'s residual claims. So, in Riordan's terminology, $1-\alpha$ is a measure of the degree of backward integration. Riordan's main result is to show the optimality of a certain degree of backward integration under "normal" circumstances. Backward integration operates as an incentive device, mitigating the opportunistic exercise of monopsonist power by the downstream firm (the buyer). It is true that equity holdings are a reality in industries like the car-manufacturing in Japan, and Riordan's paper successfully motivates their presence. But, we take a bit different viewpoint about α. Our basic setting is the rank competition among suppliers. Then, at first, according to Asanuma's empirical studies (1989), we take **a view that** α **reflects a degree of suppliers' technological initiative such that lower** α **corresponds to** *drawing supplied* **suppliers, and higher** α **does to** *drawing approved* **suppliers**. Second, α may have asymmetric, say, encouraging and discouraging incentive effects on the first period winner and loser in the second period, depending on the size of ψ, one of the incentive instruments.

Next, "Managed Competition", such as the "Japanese" form of competition in the Japanese subcontracting system or the second sourcing in the U.S Defense Procurement System, has the characteristics of the transfer of technology and knowledge after the interim ranking established at the end of the first period. Even if the competitors (two agents) have reached different capital stock configurations, learning including knowledge

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transfer or R&D spillover occurs, which may be interpreted as "Communication" or "Information sharing" among them. For example, in Japan, the various institutions (Government councils and private research circles etc.) are formed and, at the same time, private networks for the exchange of information are utilized. It is well known that in the auto/parts industry, too, the essence of the drawings contrived by one supplier may be transferred to the rival supplier. This is especially often observed in "the *drawings approved*" and/or the development activities called "Design-in". Now, let us concretely assume that when the end of first period stocks are $(\widetilde{K}_{i1}, \widetilde{K}_{j1})$, $i \neq j$, the start of the second period stock of agent i becomes

$$K_{i2}\left(\tilde{K}_{i1}, \tilde{K}_{j1}\right) = \overline{K} + \underset{\varepsilon}{E}\left(\tilde{K}_{i1} - \overline{K}\right) + t \cdot \underset{\varepsilon}{E}\left(\tilde{K}_{j1} - \overline{K}\right)$$

$= \overline{K} + h_i + t \cdot h_j$, where t is the real number, satisfying $0 \le t \le 1$. Thus, the agent can learn the rate t of skill accumulation of his competitor, and the difference between two levels of the technology of suppliers (developers) shrinks. We simply formulate this characteristic as the following assumption.

Assumption 4: The Linear Transfer Technology

The principal can offer the opportunity for transfer of knowledge and technology. Through such transfer, the difference in capital stocks decreases, and the start of second period capital stock is written as follows, given the end of first period stocks ($\widetilde{K}_{i1}$, $\widetilde{K}_{j1}$), $i \neq j$.

$$K_{i2}\left(\widetilde{K}_{i1}, \widetilde{K}_{j1}\right) = \overline{K} + \underset{\varepsilon}{E}\left(\widetilde{K}_{i1} - \overline{K}\right) + t \cdot \underset{\varepsilon}{E}\left(\widetilde{K}_{j1} - \overline{K}\right) = \overline{K} + h_i + t \cdot h_j \quad \text{for } i = 1,2, \quad i \neq j, \text{ and } 0 \le t \le 1$$

1, This assumption abstracts most simply the situation, where the principal intervenes into the process of competition among the fixed, few members, and a transfer of specific knowledge and/or technology occurs among them, for example, through the supplier associations in the automobile industry. Also in the U.S defense procurement competition, known as *second sourcing*, the developer (leader)'s technology and knowledge is transferred to the second source (the follower), who is then allowed to take over the production through accumulation of greater skill as compared to the developer.

2, This specification captures some essential ideas. The first point is that the capital accumulation is effective conditional upon the amount of accumulation $\underset{\varepsilon}{E}\left(\tilde{K}_{i1} - \overline{K}\right) = h_i, i = 1, 2$ (their own investment as a flow). The second point is the technological dependence among competitors, represented by $t \cdot \underset{\varepsilon_j}{E}\left(\tilde{K}_{j1} - \overline{K}\right) = t \cdot h_j$

for $0 \le t \le 1$. Lastly, technology transfer may be accompanied by the cost of adaptation and learning, which is formulated as $0 \le t \le 1$.

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Agents know the structure of this learning process (<u>linear</u> transfer technology) and make investments in the first period, expecting this process and the effect upon the second stage competition. The winner of the first period competition is given the favorable allotment in the production stage at the end of next period, as a reward both for the victory and for transferring his knowledge capital. In the model, this assumption implies that after the first period, the principal gives *an interim rank* to the two agents and then shrinks the difference between them *by t-fold*. That is, the difference in their capital stocks leads to $(1-t) \cdot \left(E_{\varepsilon_i}(\tilde{K}_{i1}) - E_{\varepsilon_j}(\tilde{K}_{j1}) \right) = (1-t) \cdot (h_i - h_j)$. Due to the linearity of technology transfer, the analysis becomes simpler without fundamental changes. Later, we will investigate the effect of this knowledge (technology) transfer upon the equilibrium, and the solution of the model[8].

The Payoff Functions

Here, we shall summarize the ex post payoff functions of the players in the overall game.

The payoff function of the agent i is

$$\hat{U}_i = \left\{ \alpha \cdot E_{\varepsilon_i}(\tilde{K}_{i2}) \cdot \hat{\psi} + \hat{W} - C(e_i) \right\} - g(h_i) \quad \text{for } i = 1,2, i \neq j$$

where:

$$\hat{\psi} = \begin{cases} \psi & \text{if } \tilde{K}_{i1} > \tilde{K}_{j1} \\ 1-\psi & \text{if } \tilde{K}_{i1} < \tilde{K}_{j1} \end{cases} \quad \text{and} \quad \hat{W} = \begin{cases} W & \text{if } \tilde{K}_{i2} > \tilde{K}_{j2} \\ 0 & \text{if } \tilde{K}_{i2} < \tilde{K}_{j2} \end{cases}$$

The payoff function of the principal is $\pi = (1-\alpha) \left[\psi \cdot E_{\varepsilon}(\tilde{K}_{W2}) + (1-\psi) \cdot E_{\varepsilon}(\tilde{K}_{L2}) \right] - W$.

3. The Solution of the Model.

Hereafter, we solve this three-stage game including a two-stage tournament in a backward inductive fashion. First, in the section 3.1, we analyze the second period competition *(ex post competition after the first ranking has been established.)*

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[8] Readers shouldn't consider this assumption as unimportant. The possibility of technology transfer to a second source (a loser in the first stage in the model) has not been fully developed in the existing literature (e.g. Riordan-Sappington 1989). We propose a new insight into it, especially, its total incentive effects at the ex ante and ex post stage, through the effect on both *ex ante* and *ex post*, local and global incentive constraints.

3.1   The Second Period

3.1.1   The second period tournaments with production allotments.

At the start of the second period, there exist two agents who are assigned different ranks and the ordered quantity based upon the outcome of the first period competition.   We call them the winner and the loser, respectively.   In this interim stage, the agents are not symmetric any more, different from the beginning of the first period.   It is because they start their investments given the different production allotments, as well as a possible difference in capital accumulation.[9]   Nonetheless, due to assumption 4 (spillover (transfer) of technology and knowledge), both agents have approached each other closely in terms of their skills. (Their accumulated capital stocks are 'modified'.)

According to the rank in the competition in the first period, agent i is assigned the production volume at the final production stage, as follows.

The allotment scheme is

$$q_i = \begin{cases} \psi & if & \tilde{K}_{i1} > \tilde{K}_{j1} \\ 1-\psi & if & \tilde{K}_{i1} < \tilde{K}_{j1} \end{cases} \tag{2}$$

where q_i is the instructed ordered quantity to agent i, and ψ is the production share assigned to the winner of the first period, and it satisfies $1/2 \leq \psi \leq 1$. That is, the principal can increase the ordered quantity for the winner in the first period, but this scheme is a *partially contingent allotment scheme*, which depends only on the ranking; $\tilde{K}_{i1} > \tilde{K}_{j1}$ (sub-state ω_i) or $\tilde{K}_{i1} < \tilde{K}_{j1}$ (sub-state ω_j), assigns a constant allotment over each state, and does not depend on *the difference itself* of the capital accumulation $\Delta\tilde{K}_1 = \tilde{K}_{i1} - \tilde{K}_{j1}$.[10] This is a kind of tournament scheme with contract incompleteness. We discuss the form and optimality of this allotment scheme later in the section of 4.4. We also assume that the demand curve is inelastic, and that the demand for the final goods (e.g. autos) that the principal produces is constant at the level of 1.

In addition, the two agents obtain revenues in the *ex post* bargaining from trading intermediate goods. $\tilde{K}_{W2}$

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[9] As Asanuma (1989) indicates, we suppose that the suppliers develop and exert relation specific skills, which are not identical, depending on the degree of accumulated technological maturity.

[10] Matsui (1989) *generally* investigated the cartel rule for improving the consumer surplus. His scheme is a *fully contingent* allotment scheme, which depends also on the difference in capital stocks. This paper assumes the *partially contingent* allotment scheme, but the optimal contract under local and global incentive constraints in each of two periods is explicitly investigated. The principal chooses the optimal strategies from among a set of credible contracts satisfying these constraints.

and $\widetilde{K}_{L2}$ are the final qualities, evaluated [11]by the principal, per unit of goods accumulated by the winner and the loser, until the production and sales stages.   These correspond to the valuations for the user or buyer.   By assuming zero production cost, these correspond to the gross trading value.   Since α is the bargaining power of the agent, it is also the distribution share to him of the trade gain.   Hence, $\underset{\varepsilon}{E}\left(\widetilde{K}_{W2}\right)$ and $\underset{\varepsilon}{E}\left(\widetilde{K}_{L2}\right)$ can be,

respectively, viewed as the consumer price of the final good of each kind or type (for example, imagine each type of car). $\widetilde{P}_W = \alpha \cdot \underset{\varepsilon}{E}\left(\widetilde{K}_{W2}\right)$ and $\widetilde{P}_L = \alpha \cdot \underset{\varepsilon}{E}\left(\widetilde{K}_{L2}\right)$ imply the unit revenue (input price) that the (final) winner

and the (final) loser obtain, respectively[12]. Therefore, given the quantity allotment ψ and $1-\psi$, the total revenues from the ex post bargaining for the winner and the loser are $\widetilde{P}_W \cdot \psi$ and $\widetilde{P}_L \cdot (1-\psi)$.

In this situation, they compete with each other for a common prize W, given the modified capital stocks, K_{12} and K_{22}. W is the (monetary) prize given to the final winner, whose capital is larger than that of the loser. Later, we investigate as to *why* the principal should set the monetary bonus W at the final stage, based upon the second period relative performance (rank order)[13] .

Each of the agents (the winner and the loser) incurs a cost, $C\left(e_W\right)$ and $C\left(e_L\right)$ on making the specific

investment. We have already set the assumption (A2') on the second period cost function .

The rents of the winner and the loser in the second period are

$$U_{W2} = \alpha \cdot \underset{\varepsilon}{E}\left(\widetilde{K}_{W2}\right)\cdot\psi + W \cdot I\left(\widetilde{K}_{W2} \geq \widetilde{K}_{L2}\right) - C\left(e_W\right) \qquad \text{and}$$

$$U_{L2} = \alpha \cdot \underset{\varepsilon}{E}\left(\widetilde{K}_{L2}\right)\cdot(1-\psi) + W \cdot\left(1 - I\left(\widetilde{K}_{W2} \geq \widetilde{K}_{L2}\right)\right) - C\left(e_L\right)$$

where $I\left(\widetilde{K}_{W2} \geq \widetilde{K}_{L2}\right)$ is the indicator function of the event that $\widetilde{K}_{W2} \geq \widetilde{K}_{L2}$.[14]

Let V_{W2} and V_{L2} be the expected value functions for the winner and the loser in the second period. Then,

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[11] The real qualities for the first period winner and loser are, respectively, $\underset{\varepsilon}{E}\left(\widetilde{K}_{W2}\right)$ and $\underset{\varepsilon}{E}\left(\widetilde{K}_{L2}\right)$. Remember that the noises have no real

values, and these affect *only* the principal's *ranking* decision.

[12] We do not explicitly deal with the way how the consumer price of the good is determined. But, it is easy to contrive a mechanism, which implements the price in the text in equilibrium.   Also, it does not change the essence of what I will convey to readers qualitatively, even if we consider another game that implements another part price (price of intermediate goods), for example, the bargaining between firm and consumers. Only at this stage, the prices of the goods become verifiable and so are committed.

[13] Since W is a lumpy prize, the structure of the second period competition is an *asymmetric tournament*, given a different allotment pair to the two agents.   This is a technical point different from Konishi etal. (1996).

[14] In general, $I\left(A\right)$ denotes the indicator function of an event A, where: $I\left(A\right) = \begin{cases} 1 & if\ A\ \ happens \\ 0 & otherwise \end{cases}$

$$V_{W2}\left(e_L;\Delta K;\psi,W;\alpha,t\right)= \underset{e_W}{Max}\left\{\alpha\cdot\underset{\varepsilon}{E}\left(\tilde{K}_{W2}\right)\cdot\psi +\Phi\left(\Delta K+\Delta e\right)\cdot W-C\left(e_W\right)\right\} \tag{3}$$

$$V_{L2}\left(e_W;\Delta K;1-\psi,W;\alpha,t\right)= \underset{e_L}{Max}\left\{\alpha\cdot\underset{\varepsilon}{E}\left(\tilde{K}_{L2}\right)\cdot\left(1-\psi\right)+\left\{1-\Phi\left(\Delta K+\Delta e\right)\right\}\cdot W-C\left(e_L\right)\right\} \tag{4}$$

where $\Phi\left(\Delta K+\Delta e\right)$ is the probability of winning in the second period for the winner in the former period, when the winner and the loser make the investments at the level of e_W and e_L, given ΔK (the difference in capital stocks between them: that is, $\Delta K=\left(1-t\right)\left(h_W-h_L\right)$). That is,

$$\Phi\left(\Delta K+\Delta e\right):=\text{Prob}\left(\tilde{K}_{W2}\geq\tilde{K}_{L2}\right)=\text{ Prob}\left(\Delta K+e_W-e_L>\varepsilon_L-\varepsilon_W\right)=\Phi\left(\Delta K+e_W-e_L\right)$$

where Φ is the distribution function of the random variable $\varepsilon_L-\varepsilon_W$, and we denote the density function by the small letter ϕ. We assume that ϕ is a decreasing function for positive values, i.e. $\phi'\left(\chi\right)<0$ for $\chi>0$, and that it has a finite support $\left[-\bar{\varepsilon},\bar{\varepsilon}\right]$, and is symmetric within that range, i.e., $\phi\left(\chi\right)=\phi\left(-\chi\right)$ $\forall\chi\in\left[-\bar{\varepsilon},\bar{\varepsilon}\right]$.

The mathematical representations (3) and (4) are the problems that both the winner and the loser face in the second- period. We postpone the analysis of the *global* incentive constraints of both agents until later.

Now, the first order conditions for the second period optimization problems of the agents are,

$$\alpha\psi +\frac{\partial\Phi\left(\Delta K+e_W-e_L\right)}{\partial e_W}W-C'\left(e_W\right)=0 \tag{5}$$

$$\alpha\left(1-\psi\right)-\frac{\partial\Phi\left(\Delta K+e_W-e_L\right)}{\partial e_L}W-C'\left(e_L\right)=0 \tag{6}$$

In other words, the solution of the following two simultaneous equations represents the Nash equilibrium in the second period under the assigned production allotments.

$$\alpha\psi +\phi\left(\Delta K+e_W-e_L\right)W=C'\left(e_W\right) \tag{7}$$

$$\alpha\left(1-\psi\right)+\phi\left(\Delta K+e_W-e_L\right)W=C'\left(e_L\right) \tag{8}$$

The first terms of the formulas (7) and (8) show the marginal revenue of the (ex post) capital accumulation, given the assigned ordered quantity. The second terms show the marginal value products of the second period investments, through the marginal improvement of the probability of getting the monetary payment (subsidy) W. To ensure the existence of an optimum in the second period problem, we assume that the following second order conditions are satisfied.

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$$\phi'(\Delta K + e_W - e_L)W - C''(e_W) < 0$$
$$-\phi'(\Delta K + e_W - e_L)W - C''(e_L) < 0$$

[Proposition 1]

In the equilibrium of the second period investment competition, the leading firm (the winner in the first period) never invests less than the follower (the loser in the first period).

[Proof]

In order to compare the equilibrium incentives of both agents, we combine equations (7) and (8), and get

$$C'(e_W) - C'(e_L) = (2\psi - 1) \cdot \alpha \tag{9}$$

Noting that $\alpha > 0$, $\psi \geq 1/2$, and $C'(e)$ is increasing in e, it follows that

$$e_W^* \geq e_L^* \tag{10}$$

Q.E.D.

According to the ranking based upon the outcome of the first period competition, a difference between the shares in the production at the end of second period $\psi - (1-\psi) = 2\psi - 1$ is imposed, resulting in the difference of $(2\psi - 1)\alpha$ in the marginal productivities of investment in the second period. This is why the asymmetric equilibrium arises in the second period, and from (10), the winner exerts more effort than the loser does in equilibrium (when $\psi = 1/2$, it follows that $e_W^* = e_L^*$).

Let us investigate the above results, from the viewpoint of *"strategic substitutability and complementarity"*. Differentiating (7) and (8) representing the Nash equilibrium as to e_L and e_W, respectively, we obtain the following effect on the marginal profitability.

$$-\phi'(\Delta K + e_W^* - e_L^*) \cdot W > 0 \tag{12}$$
$$\phi'(\Delta K + e_W^* - e_L^*) \cdot W < 0 \tag{13}$$

Figure 2-1 around here

That is, in the neighborhood around the second period equilibrium, the investments are *strategic complements* for the winner, and *strategic substitutes* for the loser. This implies that these two agents react differently when $1/2 < \psi < 1$. The reaction function of the leading firm (the winner) is increasing in the follower's (the loser's) effort, whereas the reaction function of the trailing firm (follower) is decreasing in the leader's (the winner's) effort. This figure suggests that the leading firm has more incentive to invest (*becomes aggressive*) when it faces

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intense competition against the follower, whereas the trailing firm has more incentive to invest when the winner (rival) becomes *less aggressive.*

3.1.2 *Comparative Statics on the second period Nash Equilibrium: The Effects of $(W, \psi), (t, \alpha)$*

Now, we obtain the some propositions on comparative statics on the second period Nash Equilibrium.

[Proposition 2]

Suppose that Assumption (A2) holds. Then, an increase in the size of the prize W induces an increase in both agents' incentives, but the <u>difference</u> between them <u>decreases</u>. In other words, the marginal incentive for investment increases larger for the loser than for the winner. That is,

$$\frac{\partial e_W^*}{\partial W} > 0, \frac{\partial e_L^*}{\partial W} > 0, \frac{\partial \left(e_W^* - e_L^* \right)}{\partial W} \leq 0$$

[Proof]

Differentiating equations (F.O.Cs) (7) and (8) and setting $d\psi = d\alpha = 0$, we obtain the following matrix representation.

$$\begin{bmatrix} \phi'W - C''(e_W) & -\phi'W \\ \phi'W & -\phi'W - C''(e_L) \end{bmatrix} \begin{bmatrix} \dfrac{\partial e_W^*}{\partial W} \\ \dfrac{\partial e_L^*}{\partial W} \end{bmatrix} = \begin{bmatrix} -\phi \\ -\phi \end{bmatrix}$$

Here, we check whether the stability condition is satisfied. Let the Hessian determinant be $|D|$. Then, $|D| = \left(\phi'W - C''(e_W) \right) \cdot \left(-\phi'W - C''(e_L) \right) + \left(\phi'W \right)^2 > 0$. The positive sign was derived from the second order conditions (S.O.Cs) and the conditions on Φ and C.

Solving the matrix systems by using Cramer's Rule, we can obtain

$$\frac{\partial e_W^*}{\partial W} = \frac{|D_1|}{|D|} = \frac{\phi \cdot C''\left(e_L^* \right)}{|D|} > 0$$

$$\frac{\partial e_L^*}{\partial W} = \frac{|D_2|}{|D|} = \frac{\phi \cdot C''\left(e_W^* \right)}{|D|} > 0$$

Considering the difference in equilibrium incentives, we have

$$\frac{\partial \left(e_W^* - e_L^* \right)}{\partial W} = \frac{|D_1| - |D_2|}{|D|} = \frac{\phi \cdot \left[C''\left(e_L^* \right) - C''\left(e_W^* \right) \right]}{|D|} \leq 0$$

This result is obtained from $C''' \geq 0$, $e_W^* \geq e_L^*$ and $|D| > 0$ **Q.E.D.**

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In order to understand this result intuitively, it is necessary to remember that the prize (subsidy) in the second period is given to the final winner, based upon the final rank irrespective of the interim rank of the capital accumulation competition. From the F.O.Cs (7) and (8), an increase of the prize W implies an increase in the marginal value products for both agents. Thus, it has a positive effect upon the second-period investments of both agents. By the way, the marginal revenue for each agent through the increase in W is the same value, $\phi\left(\Delta K + e_W^* - e_L^*\right)$. Noticing that the winner has more investments in equilibrium, $e_W^* \geq e_L^*$, the same marginal revenue induces more incentive from the loser, under the convexity of the cost function in effort incentive. We recognize from this fact that the increased subsidy in the second period shrinks the difference between the equilibrium incentives of two agents.

[Corollary 1]
We assume that $C(e) = e^2/2$. In this case, using the F.O. Cs (7) and (8), the equilibrium investment levels are

$$e_W^*(\psi, W; \Delta K, \alpha, t) = \alpha\psi + \phi^* W, \quad e_L^*(\psi, W; \Delta K, \alpha, t) = \alpha(1-\psi) + \phi^* W$$
$$\text{where } \phi^* = \phi\left(\Delta K + e_W^* - e_L^*\right) = \phi\left(\Delta K + \alpha(2\psi - 1)\right)$$

Then, by the statements of formula (11),

$$\frac{\partial e_W^*}{\partial W} = \frac{\partial e_L^*}{\partial W} = \phi^* > 0, \frac{\partial\left(e_W^* - e_L^*\right)}{\partial W} = 0$$

The implication is that, the increased prize W of the second period has a positive effect upon the equilibrium investment of each agent, but in the "quadratic" cost function case, the size of the effort increase is the same, and so, the difference between effort incentives remains unchanged. On the other hand, as proposition 2 shows, in the case of $C(e) = \frac{R}{2}e^\beta$ where $\beta > 2$ and $R > 0$, $e_W^* - e_L^*$ decreases as W increases.

Next, we examine the effect of increasing technology (information) transfer t on the second period equilibrium.

[Proposition 3]
An increase in the degree of information transfer t induces an increase in both agents' incentives, but the difference between them shrinks. In other words, the marginal incentive for investment increases larger for the loser than for the winner.

[Proof]

Differentiating F.O.Cs (7) and (8) and setting $d\alpha = d\psi = dW = 0$, we obtain the following matrix representation.

$$\begin{bmatrix} \phi'W - C''(e_W) & -\phi'W \\ \phi'W & -\phi'W - C''(e_L) \end{bmatrix} \begin{bmatrix} \dfrac{\partial e_W^*}{\partial t} \\ \dfrac{\partial e_L^*}{\partial t} \end{bmatrix} = \begin{bmatrix} (h_W - h_L)\phi'W \\ (h_W - h_L)\phi'W \end{bmatrix}$$

where h_W and h_L represent the first period investments that were provided by the winner and the loser. We have already checked the stability condition and the Hessian determinant $|D| > 0$. Solving the matrix systems by using Cramer's Rule, we can obtain

$$\frac{\partial e_W^*}{\partial t} = \frac{|D_1|}{|D|} = \frac{-(h_W - h_L)C''(e_L^*)\phi'W}{|D|} > 0$$

$$\frac{\partial e_L^*}{\partial t} = \frac{|D_2|}{|D|} = \frac{-(h_W - h_L)C''(e_W^*)\phi'W}{|D|} > 0$$

$$\frac{\partial (e_W^* - e_L^*)}{\partial t} = \frac{-(h_W - h_L)\left[C''(e_L^*) - C''(e_W^*)\right]\phi'W}{|D|} < 0$$

This result is obtained from $C''' \geq 0, e_W^* \geq e_L^*$, and $|D| > 0$, and it means that an increase in the degree of information transfer t induces an increase in both agents' incentives, but the marginal incentive for loser would be revived more, and the difference between them would shrink in the second period. This is especially effective as the initial difference at the second period $h_W - h_L$ is greater.

Q.E.D.

This shows that the technology transfer t has the same effect on the equilibrium incentives as W. As next comparative statics, we can obtain the effect of the increased ψ upon the equilibrium incentives.

[Proposition 4]

The effect of the change in ψ upon the Nash equilibrium in the second period is as follows.

$$\frac{\partial e_W^*}{\partial \psi}? \quad \frac{\partial e_L^*}{\partial \psi} < 0 \quad \frac{\partial (e_W^* - e_L^*)}{\partial \psi} > 0 \quad \frac{\partial (e_W^* + e_L^*)}{\partial \psi} < 0$$

, and $\dfrac{\partial \left[\psi e_W^* + (1-\psi)e_L^*\right]}{\partial \psi} = (e_W^* - e_L^*) + \psi \dfrac{\partial (e_W^* - e_L^*)}{\partial \psi} + \left(\dfrac{\partial e_L^*}{\partial \psi}\right)$

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[Proof]

Differentiating F.O.Cs (7) and (8) and setting $d\alpha = dt = dW = 0$, we obtain the following matrix representation.

$$\begin{bmatrix} \phi'W - C''(e_W) & -\phi'W \\ \phi'W & -\phi'W - C''(e_L) \end{bmatrix} \begin{bmatrix} \dfrac{\partial e_W^*}{\partial \psi} \\ \dfrac{\partial e_L^*}{\partial \psi} \end{bmatrix} = \begin{bmatrix} -\alpha \\ \alpha \end{bmatrix}$$

We have already checked the stability condition and the Hessian determinant $|D| > 0$. Solving the matrix systems by using Cramer's Rule, we can obtain

$$\frac{\partial e_W^*}{\partial \psi} = \frac{\alpha \left[2\phi'W + C''(e_L) \right]}{|D|}?, \qquad \frac{\partial e_L^*}{\partial \psi} = \frac{\alpha \left[2\phi'W - C''(e_W) \right]}{|D|} < 0$$

$$\frac{\partial \left(e_W^* - e_L^* \right)}{\partial \psi} = \frac{\alpha \left[C''(e_W^*) + C''(e_L^*) \right]}{|D|} > 0$$

$$\frac{\partial \left(e_W^* + e_L^* \right)}{\partial \psi} = \frac{\alpha \left(4\phi'W - \left[C''(e_W^*) - C''(e_L^*) \right] \right)}{|D|} < 0$$

where S.O.C s: $\phi'W - C''(e_W) < 0$ and $-\phi'W - C''(e_L) < 0$ and $C''' \geq 0$ are used when four signs are derived.

Q.E.D.

This implies that an increase of ψ (the volume of allotment assigned to the winner) discourages the loser greatly after its implementation, thereby the <u>difference</u> between equilibrium incentives <u>increases</u>. This is in contrast with the results on the effect of W. As for the winner's incentive, it's a bit involved. The increase of the volume of assigned allotment generally increases the investment incentives holding the other's investment fixed. However, when both the winner and the loser invest, shifting (transferring) more ψ from the loser to the winner moves the winner's best response function $e_W^* (e_L; \psi, W)$ outward, but at the same time, moves the loser's best response curve $e_L^* (e_W; \psi, W)$ downward (inward). Thus, <u>the Milgrom-Roberts Theorem (1990)</u> *cannot* be applied. As the computation result shows, we don't know for sure that the equilibrium incentive $e_W^* (\psi, W)$ by the winner increases or not, since *feedback* from decreased e_L can be so strong as to override the direct effect by the increase of ψ.[15] (Nonetheless, we can say for sure that the equilibrium e_L^* goes down, because the direct effect and the feedback effect work in the same direction of decreasing e_L^*.) As for these contrastive incentive

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[15] By interpreting $(\psi, 1 - \psi)$ as an asset ownership structure, we see that this comparative statics result on the effect of increasing ψ is rather similar to the one on the change of asset ownership in Hart-Moore (1990).

effects of W and ψ, compare the two figures 2-2 and 2-3.

Figure 2-2 and 2-3 around here

The last term in the proposition 4 represents how the weighted average $\psi e_W^* + (1-\psi) e_L^*$ of the equilibrium incentives is affected by the marginal change in ψ. The sign is ambiguous because the first and second terms are positive, but the third term is negative. Then,

[Proposition 5]

An increase in the bargaining power α of each agent unambiguously increases the second period equilibrium incentive of the winner, but its effect is ambiguous for the second period equilibrium incentive of the loser.

[Proof]

Differentiating F.O.Cs (7) and (8) and setting $dt = d\psi = dW = 0$, we obtain the following matrix representation.

$$\begin{bmatrix} \phi'W - C''(e_W) & -\phi'W \\ \phi'W & -\phi'W - C''(e_L) \end{bmatrix} \begin{bmatrix} \dfrac{\partial e_W^*}{\partial \alpha} \\ \dfrac{\partial e_L^*}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} -\psi \\ -(1-\psi) \end{bmatrix}$$

We have already checked the stability condition and the Hessian determinant $|D| > 0$. Solving the matrix systems by using Cramer's Rule, we can obtain

$$\frac{\partial e_W^*}{\partial \alpha} = \frac{\psi\left(\phi'W + C''(e_L)\right) + (\psi - 1)\phi'W}{|D|} > 0$$

$$\frac{\partial e_L^*}{\partial \alpha} = \frac{(2\psi - 1)\phi'W + (1-\psi)C''(e_W^*)}{|D|} \quad ?$$

This result implies that an increase in α might have an *asymmetric incentive effect* for the winner and the loser in equilibrium. Indeed, when ψ is close to 1, the best response function of the winner shifts outward even more than the loser's one does. Then, $\dfrac{\partial e_L^*}{\partial \alpha} < 0$ holds in equilibrium. This is the case, where the positive direct effect on the loser's incentive e_L^* by the increase of α is completely overridden by the negative *feedback* from increased e_W.[16]

3.1.3 Summery

Summarizing the above analysis, the expected profits to be obtained in equilibrium by both the winner and

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[16] Notice that e_L and e_W are *strategic substitutes* for the loser.

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the loser are as follows.

$$V_{W2}^* = \alpha \left(K_{i2} + e_W^* \right) \psi + \Phi^* W - C \left(e_W^* \right) \tag{14}$$

$$V_{L2}^* = \alpha \left(K_{i2} + e_L^* \right) (1 - \psi) + (1 - \Phi^*) W - C \left(e_L^* \right) \tag{15}$$

where $\Phi^* = \Phi(\Delta K + e_W^* - e_L^*) \geq 1/2$, $K_{i2} = \overline{K} + h_i + t \cdot h_j$.

As for the second term, recall that by the assumption, the noises ε_i and ε_j have no lasting value into the second period. Also, each of e_W^* and e_L^* is a function of ΔK and α, t, given ψ, W.

Now, the difference between the equilibrium profits is,

$$\Delta V^* \left(K_{i2}; \psi, W; \alpha, t \right) = \alpha \left(2\psi - 1 \right) K_{i2} + \left\{ \alpha [\psi e_W^* - (1 - \psi) e_L^*] + \left(2\Phi^* - 1 \right) W - \left[C \left(e_W^* \right) - C \left(e_L^* \right) \right] \right\} \tag{16}$$

This is *the discrete prize*, which positively induces the ex ante (first period) incentives from the agents when $\psi > 1/2$, and this prize establishes the *ex ante* competition. Let us summarize the mechanism, which works so far in the model. The principal announces and commits herself to an incentive scheme, where the principal evaluates agents based upon the interim ranking (relative performance) of the outcome of the first period competition, and then changes the production share favorable to the winner *discretely*. Thereby, when the two agents (the leader and the follower) compete for the monetary prize in the second period, there exists an asymmetric (pure strategy) equilibrium due to the difference in the marginal productivities, resulting in a difference between the equilibrium profits of the two agents. This in turn works as a *carrot (prize)*, which enhances the ex ante incentives of the two agents. Therefore, when $\psi > 1/2$, the agents are faced with the tournament scheme with the prize of (16), where the reward is *discontinuously* based upon his performance (see Figure 3). This is the essential logic of the analysis so far. The global incentive constraint in the second period (interim individually rationality constraint) will be investigated later.

Figure 3 around here

3.2   The first period

At the start of the 0-period, the principal is faced with homogeneous agents with the capital stock level $\overline{K}$. She announces an organizational structure (incentive scheme), concretely, the rule for the production allotment in the final period and the rule of competition for the monetary prize. The former is related to whether

the principal *endogenously* chooses a non-elimination tournament $\psi \neq 1$, where the loser is not eliminated and given the one more chance in the second period, or the elimination tournament $\psi = 1$, where the loser is made to drop out of the race in the second period.

  <u>After accepting the contract,</u> two agents choose the capital investment levels of the first period, so as to maximize their own expected payoffs. Now, we suppose that the capital stock of agent i at the end of the first period is $\tilde{K}_{i1} = \overline{K} + h_i + \varepsilon_i, \quad i = 1, 2$. $\overline{K}$ is the capital stock at the start point, observable among the principal and the two agents. h_i is the investment level during the first period. ε_i is the uncertainty factor (the random shock in the first period, with no lasting value). Next, we suppose that $F(h)$, where $h = h_i - h_j$, is the probability of agent i winning in the first period competition. Then,

$$F = Prob(\tilde{K}_{i1} > \tilde{K}_{j1}) = Prob(h_i - h_j > \varepsilon_j - \varepsilon_i) = F(h_i - h_j) \tag{18}$$

$\varepsilon_j - \varepsilon_i$ is the difference between the noise ε_j and ε_i and F is the distribution function of the random variable $\varepsilon_j - \varepsilon_i$. From the analysis in the former section, we see that the two agents are faced with <u>the following tournament scheme.</u>

$$S_i\left(\tilde{K}_{i1,}\tilde{K}_{j1}\right) = \begin{cases} V_{L2}^* & if & \tilde{K}_{i1} < \tilde{K}_{j1} \\ V_{L2}^* + \Delta V^* & if & \tilde{K}_{i1} > \tilde{K}_{j1} \end{cases}$$

If the agent wins the race in the first period, he can obtain *the discrete prize* ΔV^* in addition to the base value V_{L2}^*. The reward to be obtained by agent i is based upon his absolute performance $\tilde{K}_{i1}$ *discontinuously*. See Figure 3. This is different from the relative performance scheme where the reward is based upon his absolute performance $\tilde{K}_{i1}$ *continuously*. The important point of this scheme is <u>the discontinuity</u>, rather than <u>the nonconcavity</u>, because it generates the basis for establishing the ex ante fierce competition between agents.

 The two agents solve the following problems simultaneously and independently, given the rival's investment h_j.

$$\max_{h_i} E_\varepsilon\left\{F(h)\cdot V_{W2}^* + \left(1 - F(h)\right)\cdot V_{L2}^*\right\} - g(h_i) \qquad i = 1, 2, i \neq j \tag{19}$$

The fundamental equation of this problem, given h_j, is

$$V_i(\overline{K}, \overline{K}, h_j) = \max_{h_i}\left\{V_{L2}^* + F(h)\cdot \Delta V^*\right\} - g(h_i) \qquad i = 1, 2, i \neq j \tag{20}$$

 The first term V_{L2}^* represents the value, which the loser gets in the Nash equilibrium in the second period, when she loses in the first period competition, given $\Delta K, \psi$ and W (the principal's two instruments). The second term $F(h)\cdot \Delta V^*$ is the expected prize, which implies that with the probability of $F(h)$, the agent can get this lumpy prize ΔV^*, which is endogenously generated <u>as the equilibrium payoff difference in the second</u>

<u>period</u> through the incentive scheme ψ and W. g (h_i) is the cost function of investments h_i in the relation specific skills, with $g' > 0$, $g'' \geq 0$. Now, we shall define $\bar{\upsilon}$ and $\underline{\upsilon}$ as follows.

$$\bar{\upsilon} := \bar{\upsilon}\left(\psi, W; \alpha, t\right) = \left(\alpha \cdot e_W^*\right) \cdot \psi + \Phi^* \cdot W - C\left(e_W^*\right)$$

$$\underline{\upsilon} := \underline{\upsilon}\left(\psi, W; \alpha, t\right) = \left(\alpha \cdot e_L^*\right) \cdot \left(1 - \psi\right) + \left(1 - \Phi^*\right) \cdot W - C\left(e_L^*\right)$$

where $\Phi^* := \Phi\left(\Delta K + e_W^* - e_L^*\right) \geq 1/2$ (the equality is satisfied when $\psi = 1/2$ and $\Delta K = \left(1 - t\right)\left(h_W - h_L\right) = 0$)

Solving the problem, we can obtain the F.O.C for agent 1.

$$\alpha\left(1 - \psi\right) \cdot 1 + F\left(h_1 - h_2\right)\left[\left(2\psi - 1\right) \cdot \alpha \cdot 1 + 2\left(1 - t\right)\phi\left(\Delta K + e_W^* - e_L^*\right)W\right]$$
$$+ \left[\left(2\psi - 1\right)\alpha K_{12} + \left(\bar{\upsilon} - \underline{\upsilon}\right)\right]f\left(h_1 - h_2\right) = g'\left(h_1\right) \tag{21}$$

where $K_{12} = \bar{K} + h_1 + t \cdot h_2$ implies that the RHS (right hand side) consisting of the initial stock $\bar{K}$ plus the capital (skill) accumulation h_1 by agent 1 plus the rate of t of the accumulation h_2 by the rival suppliers lead to the modified stock level K_{12} (LHS) at the start of period 2.

The equation (21) implicitly defines the reaction function of agent 1 in the first stage. Similar conditions can be obtained with respect to agent 2. Since both agents have identical skills at the beginning of the first period and so they are identical, in a symmetric equilibrium, the incentives are $h_1^* = h_2^* = h\left(\psi, W; \alpha, t\right)$

In this case, the first order conditions are simplified as follows, which characterize the symmetric subgame perfect Nash equilibrium investment level.

$$\alpha\left(1 - \psi\right) + F\left(0\right) \cdot \left[\left(2\psi - 1\right)\alpha + 2\left(1 - t\right)\phi\left(e_W^* - e_L^*\right)W\right]$$
$$+ \left[\left(2\psi - 1\right)\alpha\left(\bar{K} + \left(1 + t\right)h^*\right) + \bar{\upsilon} - \underline{\upsilon}\right]f(0) = g'\left(h^*\right) \tag{22}$$

The three terms in the Left Hand Side (LHS) represent the following effects. The first term represents the marginal increase in the value V_{L2}^* (formula (15)) that is expected to be obtained in the second period Nash equilibrium, when he loses in the first period competition (race), through the increase in capital investment in the first period. The second term represents the marginal increase in the discrete prize: $V_{W2}^* - V_{L2}^*$ itself, with the probability of winning being $F\left(0\right) = 1/2$ in equilibrium. The two terms in the bracket $\left[\ \right]$ of the second term are, respectively, *the direct effect* and *the marginal strategic effect*. The direct effect means the (private) marginal revenue from the increase in the ordered quantity $\left(2\psi - 1\right)$, with equal probability in equilibrium. The marginal strategic effect $F\left(0\right) \times 2\left(1 - t\right)\phi\left(e_W^* - e_L^*\right)W = \left(1 - t\right)\phi\left(e_W^* - e_L^*\right)W$ implies a strategic incentive for the agents to increase marginally <u>the probability of winning in the final period</u> through increasing the difference in the first period capital accumulation. The third term is *the tournament effect* through marginal improvement of the

probability of winning, given the equilibrium payoff difference, that is, the discrete prize. By summing up the two direct effects, the F.O.Cs (Local Incentive Constraints) are transformed as follows.

$$\frac{1}{2}\alpha + (1-t)\phi\left(e_W^* - e_L^*\right)W + \left[(2\psi - 1)\alpha\left(\bar{K} + (1+t)h^*\right) + \bar{\upsilon} - \underline{\upsilon}\right]f(0) = g'(h^*) \tag{22}'$$

Further, in this two-stage game, the global incentive constraint must be satisfied in the ex ante stage in the first period symmetric equilibrium. The agents can choose an alternative to deviate from the intense competition, becoming contented with the position of the loser in the tournaments, and so they can get the following intertemporal payoff.[17]

$$U = \max_{h_i}\left\{V_{L2}^* - g(h_i)\right\} = \left\{\alpha(1-\psi)\left(\bar{K} + h_L + th^*\right) + \underline{\upsilon} - g(h_L)\right\} \tag{23}$$

where h_L is the maximand of the above formula. Each agent can secure at least the payoff of $\underline{\upsilon}$ by choosing the level of h_L when the rival may choose h^*. Now, let us check the global incentive constraint in the first period.[18]

$$\frac{1}{2}\left[V_{W2}^*\left(h^*, h^*\right) + V_{L2}^*\left(h^*, h^*\right)\right] - g(h^*) \geq U \tag{24}$$

$$\Leftrightarrow \frac{\alpha}{2}\left\{\bar{K} + (1+t)h^* + \left[\psi e_W^* + (1-\psi)e_L^*\right]\right\} + \frac{1}{2}W - \frac{1}{2}\left[C(e_W^*) + C(e_L^*)\right] - g(h^*)$$

$$\geq U = \left\{\alpha(1-\psi)\left(\bar{K} + h_L + th^*\right) + \underline{\upsilon}\right\} - g(h_L)$$

$$= \left\{\alpha(1-\psi)\left(\bar{K} + h_L + th^* + e_L^*\right) + \left(1 - \Phi\left(\Delta K + e_W^* - e_L^*\right)\right)W - C(e_L^*)\right\} - g(h_L)$$

Putting this inequality in order, we obtain

$$\frac{\alpha}{2}\left[\bar{K} + (1+t)h^*\right] + \frac{1}{2}\left[\bar{\upsilon} + \underline{\upsilon}\right] - g(h^*) \geq \alpha(1-\psi)\left(\bar{K} + h_L + th^*\right) + \underline{\upsilon} - g(h_L) \tag{25}$$

Hence, we obtain the following proposition 4 about the (ex ante) global incentive constraint.

[Proposition 6]: The Global Incentive Constraints in the first period.

There exists a symmetric equilibrium above the level of h_L in the first period only if

$$\frac{\alpha}{2}\cdot\left(\bar{K} + (1+t)h^*\right) + \frac{1}{2}(\bar{\upsilon} - \underline{\upsilon}) - \alpha(1-\psi)\left(\bar{K} + h_L + th^*\right) \geq g(h^*) - g(h_L) \tag{26}$$

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[17] We assume that the random error is not a large shock, so that a firm cannot win *only through luckiness* when it exerts the level h_L and the rival exerts the level h^*. This is consistent with the other assumptions and results of the model.

[18] The ex ante individual rationality constraint is $(1/2)\left[V_{W2}^* + V_{L2}^*\right] - g(h^*) \geq 0$. When the above global incentive constraint is satisfied, this inequality is also satisfied. That is, we suppose that U is strictly positive.

The first term in the bracket of the LHS of (26); $\frac{\alpha}{2} \cdot \left(\bar{K} + (1+t)h^* \right)$ represents the expected revenue to be

obtained from the first period capital accumulation, because the agents can get the one-half of the total allotment 1 in expectation, under which he is accumulating the capital $\bar{K} + (1+t)h^*$ in the first period per unit of the

allotment and gets the share α of it, due to his bargaining power. The second term in the brackets is the expected

prize given the probability of winning in equilibrium (the payoff spread $(\bar{\upsilon} - \underline{\upsilon})$ generated in the asymmetric

equilibrium in the second period, due to the policy on production allotment to the agents).   The right hand side

(RHS) is the extra cost when he chooses the equilibrium investment level h^*, not the default level h_L, given the

rival's investment behavior of choosing h^*.

We can interpret this proposition 6 clearly, including the interpretation of the third term of the LHS, in terms

of the incentive constraint in finitely repeated games[19].   In the LHS, the second term shows that the agent can

obtain the prize of $\bar{\upsilon} - \underline{\upsilon}$ with probability $1/2$, if he plays the equilibrium level h^*, when the opponent (rival)

plays h^*. Nonetheless, if he deviates from h^* to h_L, he loses with certainty, obtaining the allotment $1 - \psi$, under

which he gets the share α of the total quality (gross gain from trade), consisting of the initial stock $\bar{K}$, his

investment h_L generated through the first period capital accumulation, and t times the rival's larger investment

h^*. If the agent had invested the equilibrium level of h^*, he would have obtained the larger benefit from his own

capital accumulation itself through getting the larger allotment, even with the probability of one-half.

The sum of these three terms is _the continuation loss_ of deviating from h^* to h_L in the first period. It is a

'penalty' imposed upon first period shirking (deviation) in terms of the second period payoff. On the other hand,

the RHS represents the cost saving of the investment due to the deviation ('shirking' or 'cheating').   This is _the_

deviation incentive.   Therefore, if the continuation loss is larger than the deviation incentive, then the investment

level h^* can be supported as a subgame perfect equilibrium in the first period.

Now, when the condition (26) is satisfied, we can compare the local (marginal) incentive in the first period

on the equilibrium path with that in the case where only one agent is the trading partner.   In the case of one

agent ('bilateral monopoly'), the agent is given the rate α of the end of second-period capital stock $\tilde{K}_2$

represented by (1), which is equivalent to the gross total value of trade.   In other words, the agent considers his

bargaining power equivalent of the capital stock $\tilde{K}_2$, as private revenue, when he decides the first and second

period investments.   His objective function at the beginning of the first period can be described as

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[19] See, for example, Benoit, P and V, Krishna (1985).

$$E_\varepsilon\left[\alpha \cdot \tilde{K}_2 - C(e)\right] - g(h) \tag{27}$$

He chooses the first period and the second period investment levels, so as to maximize the formula (27). The equilibrium investment level h_S and e_S satisfy the following F.O.Cs.

$$\alpha = C'(e_S) \tag{28}$$

$$\alpha = g'(h_S) \tag{29}$$

From these, we can recognize the following facts. First, the agent underinvests in the view of the whole organization, because he can get only a small part α of the value added which he generates through his investment. This is the so-called *"Hold up problem"* (more generically, the Free Rider Problem). In the case where two agents compete over two periods, the equilibrium incentive in the first period is characterized by the first order condition (22'). Comparing the F.O.Cs in the two cases, first, there exists a difference in the size in the direct effect of capital accumulation,

$$\alpha - \frac{1}{2}\alpha = \frac{1}{2}\alpha \tag{30}$$

Next, let us consider the marginal strategic effect $(1-t)\phi\left(e_W^* - e_L^*\right)W$. It implies an incentive for the agents to increase marginally the probability of winning in the final period through increasing marginally the difference in the end of the first period capital stocks as compared to the rival agent. This effect does not exist in the one supplier case (monopoly case). The tournament effect has the following implication. In the case of the competition for the allotment in the production "cartel", the allotment in the final stage (production and sales stage) is based upon the ranking (the rank order) of the capital accumulation in the first period. Under such an allotment mechanism, when an agent achieves larger capital accumulation and wins the race, he obtains a relatively large allotment that implies a favorable position for the ex post competition in the second period, whereby he can obtain the additional expected profit in equilibrium. He expects this prize rationally, and will compete with his rival "head-to-head" so as to increase the probability of getting it. The indirect incentive effect resulting from this behavior is *the "tournament" effect.* We compare the relative size of these three effects, obtaining the following proposition on the ex ante marginal (local) incentive in equilibrium.

[Proposition 7]: Comparison between marginal incentives:

Suppose that there exists a unique symmetric equilibrium in the first period, i.e., the inequality (26) in proposition 4 as well as the local conditions (22) is satisfied. Then, the equilibrium incentive h^* is larger than h_S (the investment level in the one supplier case) if and only if

$$(1-t)\phi\left(e_W^* - e_L^*\right)W + f(0)\left\{\alpha\left[2\psi - 1\right]\left[\overline{K} + (1+t)h_s\right] + (\overline{\upsilon} - \underline{\upsilon})\right\} \geq \frac{1}{2}\alpha \qquad (31)$$

{Marginal Strategic Effect}   {Discrete Tournament Effect}   {Direct Effect (Negative)}

The important element is *the tournament effect* as the second term of the LHS, which consists of the components.   One is the term $f(0)$, which implies the marginal improvement of the probability of winning at the symmetric equilibrium.   The other is the size of the prize (payoff difference) generated in the asymmetric equilibrium, corresponding to the terms in the brackets {}.   The first term in the brackets, $\alpha\left[2\psi - 1\right]\left[\overline{K} + (1+t)h_s\right]$ is the difference in the revenue evaluated at h_s, resulting from part of the first period capital accumulation, based upon the difference in the rank-order in the first period competition. $(2\psi - 1)$ represents the difference in the assigned allotments.   Due to this, the winner can get more revenue, even with equal equilibrium incentives.   The second term $\overline{\upsilon} - \underline{\upsilon}$ represents the payoff difference in the second period between the winner and the loser.   Of course, the marginal strategic effect $(1-t)\phi\left(e_W^* - e_L^*\right)W$ is positive for $0 \leq t < 1$.   On the other hand, the RHS is the direct effect (30), which implies that the competition has a negative incentive effect from this point of view.   Here we can recognize the following facts.   First, as $f(0)$ is larger, that is, as the support of uncertainty at the end of the first period becomes smaller, this inequality tends to be satisfied.   In addition, as the discrete prize is larger in equilibrium, investment over the hold up level defined by (29) tends to be induced.   Nonetheless, note that it is another problem whether the principal really has an incentive to induce the above incentive.

Figure 4 around here

3.3 The 0 period: The Principal's Problem

Last, we focus on the behavior of the principal.   This is the problem concerning the optimal contract design for her to maximize her private profit.   At the beginning of the 0 period, the principal chooses both ψ (the production share assigned to the first period winner) and W (the monetary reward given to the final winner), and commits herself to the two strategies.   The indirect payoff function[20] (as her objective) is,

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[20] The expected payoff that she can obtain in equilibrium of the continuation game played by the two agents, given her strategies $\{\psi, W\}$.

$$\pi^*(\psi, W) = (1-\alpha)\left\{\left(\bar{K} + (1+t)h^*\right) + [\psi \cdot e_W^* + (1-\psi) \cdot e_L^*]\right\} - W \tag{32}$$

This formula implies the principal's share of the total value generated by the induced incentives, minus her fixed cost (monetary prize). After all, the principal's problem is to maximize the expected payoff $\pi^*(\psi, W)$ defined as a function of both ψ and W, subject to the inequality constraints $1/2 \leq \psi \leq 1$ and $W \geq 0$, the global incentive constraints (26) and $V_{L2}^* \geq 0$, corresponding to the ex ante and interim (ex post) participation constraints for the agents. Hence, the overall problem is formulated such that the principal chooses the optimal (ψ, W) to maximize her expected payoff in period 0 from the set of credible contracts that satisfies these incentive constraints.$^\blacklozenge$

[Problem]

$$\textbf{Max} \qquad \pi(\psi, W) := (1-\alpha)\left[\psi \cdot \underset{\varepsilon}{E}\left(\tilde{K}_{W2}\right) + (1-\psi) \cdot \underset{\varepsilon}{E}\left(\tilde{K}_{L2}\right)\right] - W$$

{ψ , W}

s.t・ (26) the global incentive constraint for the agents in the perfect equilibrium in the first period.

・$V_{L2}^* \geq 0 \cdots$ the participation constraint for the loser in the ex post competition.

・(5),(6) $\cdots$ the local incentive constraints in the second period equilibrium.

・(22)---the local incentive constraints in the first period symmetric equilibrium.

・$1/2 \leq \psi \leq 1, \quad W \geq 0$

The Lagrangean for the problem is written as follows.

$$L = \pi^*(\psi, W) + \mu_1 \cdot \Delta_1 + \mu_2 \cdot V_{L2}^*$$

with the additional constraints, $1/2 \leq \psi \leq 1, W \geq 0$, where μ_1, μ_2 are the non-negative Kuhn Tucker Multipliers. For the notational simplicity, we incorporate the local incentive constraints in the first and second period into the constrained objective function for the principal (that is, the Lagrangean). $(\pi^*(\psi, W)$ is, as the

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$^\blacklozenge$ We formulate the model of "Managed Competition" as a Kuhn-Tucker problem, with several incentive constraints. Garcea, M.C (1993) does not explicitly investigate this theoretical question, and so he does not obtain the important theoretical implications that we have obtained from the analysis, such as the information structure and the incentives among the agents, the effect of the relaxation and/or tightening of the constraints, and the effect of the information sharing (technology transfer: t) on the marginal strategic effect. Hence, we can also understand that this paper presents a more convincing model of ' Vertical Keiretsu' in the sense that it explicitly incorporates into the analysis the ' *costs of governance*' in the form of the ' *control cost* ' analyzed below, as well as the benefits of relation specific investment.

formula (32) shows, the profit expected to be obtained by the principal on the equilibrium path, for which the equilibrium incentives in the first and second period are substituted.)   And Δ_1 is *the slack* associated with the first period global incentive constraint, and it equals the LHS minus the RHS of formula (26). Now, let us consider the first order conditions for the optimum.

3.3.1. The Conditions for ψ

First, as for ψ (the production allotment share), we get

$$\frac{\partial L}{\partial \psi} = (1-\alpha) \left\{ 2(1+t) \left[\frac{\partial h^*}{\partial \psi} \right] + \left[\left(e_W^* - e_L^* \right) + \frac{\partial e_L^*}{\partial \psi} + \psi \cdot \frac{\partial \left(e_W^* - e_L^* \right)}{\partial \psi} \right] \right\}$$
$$+ \mu_1 \cdot \left[\frac{\partial \Delta_1}{\partial \psi} \right] + \mu_2 \cdot \left[\frac{\partial V_{L2}^*}{\partial \psi} \right] \leq 0 \qquad (33)$$

$$\left(\psi - \frac{1}{2} \right) \cdot \frac{\partial L}{\partial \psi} = 0, \quad (1-\psi) \cdot \frac{\partial L}{\partial \psi} = 0 \qquad (33')$$

where μ_1 and μ_2 are the non-negative Kuhn Tucker Multipliers associated with the ex ante and ex post global incentive constraints. (33') are so called *complementary slackness conditions*.

As for (33), we can check that

$$\frac{\partial \Delta_1}{\partial \psi} = \alpha \left\{ \frac{1}{2} \left(e_W^* + e_L^* \right) + \left(\bar{K} + h_L + t h^* \right) \right\} > 0$$

Hence, as ψ increases, the global incentive constraint for the first period competition tends to be *relaxed*.   One of the reasons is the marginal increase in size of the expected equilibrium prize $: \frac{1}{2} (\bar{\upsilon} - \underline{\upsilon})$ in the second period, and the other is the decrease in the equilibrium revenue when he would lose in the first period competition and would be, as a result, assigned the allotment $1 - \psi$. This implies that the penalty becomes severe.   These two effects induce the agents to participate in the first period competition or weaken their incentives to deviate (cheat) to the level of h_L.

   Also, we see that $\frac{\partial V_{L2}^*}{\partial \psi} = -\alpha \left\{ \bar{K} + (1+t) h^* + e_L^* \right\} < 0$

   Hence, as ψ increases, the global incentive constraint for the first period loser for the second period competition tends to be *tight*. In other words, the loser tends to exit (quit) from the ex post competition.   Last, the effect of the increase in ψ upon the equilibrium surplus of the principal is

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$$\frac{\partial \pi^*}{\partial \psi} = (1-\alpha)\left\{ 2(1+t)\left[\frac{\partial h^*}{\partial \psi}\right] + \left[\left(e_W^* - e_L^*\right) + \frac{\partial e_L^*}{\partial \psi} + \psi \cdot \frac{\partial \left(e_W^* - e_L^*\right)}{\partial \psi}\right]\right\}. \tag{33'}$$

The first term in the brackets of (33'): $2(1+t)\left\{\dfrac{\partial h^*}{\partial \psi}\right\}$ is the effect, which enhances the accumulated quality

through an increase in the *ex ante* incentives through an increase in ψ. Differentiating F.O.C (22), and arranging

it, and evaluating at $h_1 = h_2 = h^*$, we get

$$\frac{\partial h^*}{\partial \psi} = \frac{-\left(\dfrac{\partial \Delta \upsilon}{\partial \psi}\right) f(0)}{\left\{(2\psi - 1)\alpha(1+t)f(0) - g''\left(h^*\right)\right\}}, \text{ where } \Delta \upsilon := (\bar{\upsilon} - \underline{\upsilon})$$

The denominator is negative because we assume that the local second order condition with respect to the first

period investment incentives can be satisfied.   Thus, the sign of the RHS of the above equation depends upon

the sign of the numerator $\left(\dfrac{\partial \Delta \upsilon}{\partial \psi}\right)$.   The value is $\alpha\left(e_W^* + e_L^*\right) > 0$, and so, $\left(\dfrac{\partial h^*}{\partial \psi}\right)$ is unambiguously positive.

This implies that, as ψ increases, the size of prize $\Delta \upsilon$ on the equilibrium path, where the winning agent

chooses the level e_W^* and the losing agent chooses the level e_L^*, is enhanced, and that the marginal increase in

this prize has a positive indirect effect upon the first period incentive.   The item in the second bracket is the

effect that leads to the increase in the ex post (second period) incentive.   First, from proposition

$1, e_W^* - e_L^* \geq 0$ (the inequality holds when $\psi > 1/2$). Next, $\left(\dfrac{\partial e_L^*}{\partial \psi}\right)$ and $\left(\dfrac{\partial \left(e_W^* - e_L^*\right)}{\partial \psi}\right)$ are each

unambiguously negative and positive, respectively.

   Hence, the increase in ψ implies a subsidy (tax) to the winner (loser), and so it has a positive effect upon the

ex ante capital accumulation but a large negative effect upon the ex post incentive of the loser. In other words, the

increase in ψ discourages the loser largely in the ex post stage.   In addition, you can notice that the explicit

cost term doesn't appear in this equation.   In the traditional tournaments, the principal needs to spend the

monetary cost (prize) in order to provide the incentives, while it doesn't appear in (33).   This provides a hint as

to why the principal may use a *nonmonetary* incentive scheme, such as "the assignment of allotments". This is

clear from the first order condition about the monetary prize W.[21]

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[21] We can interpret this prize as a kind of 'efficiency wages' for inducing the dynamic incentives. As for it, see, e.g., Shapiro and Stiglitz

(1984), and Milgrom and Roberts (1992).

3.3.2. The Conditions for W

The first order (Kuhn-Tucker) condition for the optimum for W is,

$$\frac{\partial L}{\partial W} = (1-\alpha)\left\{2(1+t)\cdot\left[\frac{\partial h^*}{\partial W}\right]+\left[\frac{\partial e_L^*}{\partial W}+\psi\cdot\frac{\partial(e_W^*-e_L^*)}{\partial W}\right]\right\}-1$$

(34)

$$+\mu_1\cdot\left[\frac{\partial\Delta_1}{\partial W}\right]+\mu_2\cdot\left[\frac{\partial V_{L2}^*}{\partial W}\right]\leq 0$$

$$W\frac{\partial L}{\partial W}=0$$

(34')

where μ_1,μ_2 are the non-negative Kuhn Tucker Multipliers, and (34') is the *complementary slackness condition* on W. In (34), the first term in the first large bracket {} is the marginal benefit to be obtained by the principal in the first period through a marginal increase in W. The second term $\left[\frac{\partial e_L^*}{\partial W}+\psi\cdot\frac{\partial(e_W^*-e_L^*)}{\partial W}\right]$

within the first large bracket {} is the marginal benefit to be obtained in the second period through increasing W. Also, the increase in W is accompanied by a direct marginal cost equal to one. When we consider the implications of these first order conditions, the noticeable points are two-fold. The first point is concerning the first term in the first brackets {}.

Differentiating F.O.C (22) with respect to W, arranging it, and evaluating at $h_1 = h_2 = h^*$, we get

$$\frac{\partial h^*}{\partial W}=\frac{-(2\Phi^*-1)\cdot f(0)}{(2\psi-1)\alpha(1+t)f(0)-g''(h^*)}$$

Since the denominator is negative due to the fact that the S.O.C are locally satisfied in the neighborhood of h*, the sign of the above equation depends upon the sign of the numerator.

When $\psi > 1/2$, then $e_W^* > e_L^*$ and $2\Phi(\Delta K + e_W^* - e_L^*)-1 > 0$. Thus, the sign of the total effect is positive.

By the way, the increase in W induces incentives in the cases of both the winner and the loser in the second period. However, it was shown from proposition 2 that the spread of incentives declines because the loser has more incentive than the winner in the second period. If so, the incentives of both agents in the first period seem to decrease, because the future prize (reward) which will be obtained through winning in the first period competition seems to become smaller, since the loser rechallenges the winner severely in the second period. However, noticing that e_W^* and e_L^* are chosen optimally in the Nash equilibrium in the second period, the total effect on the second period prize through a marginal change in e_W^* and e_L^* disappears, as is evident from the *envelope theorem*. On the other hand, the direct effect of increasing efficiency wages

$\dfrac{\partial \Delta V}{\partial W} = 2\Phi\left(\Delta K + e_W^* - e_L^*\right) - 1 > 0$ remains. Hence, there remains the effect that the incentives in the first

period are induced through the increase in size of the second period prize by increasing W. From (34), we see

that the marginal increase in W costs one unit explicitly. This implies that even though the increase in W

induces investment incentives, it is costly in terms of the *incentive cost*.

Also, we can check easily that $\dfrac{\partial \Delta_1}{\partial W} > 0$ and $\dfrac{\partial V_{L2}^*}{\partial W} > 0$.

These inequalities show that if the principal increases W, the global incentive constraints tend to be *relaxed* or

satisfied in the first period and the second period equilibrium, and this fact intuitively has a very reasonable

implication that an increase in W enhances the fascination with the competition directly, and so makes the agents

willing to participate in it.

3.3.3. Classifying the above Kuhn-Tucker conditions into two cases.

3.3.3.1. Inner Solution Cases: $1/2 < \psi < 1, \quad W > 0$

In this inner solution cases, there are further the following three cases.

$(a)\,\mu_1 > 0, \mu_2 = 0$ (Only the first period global incentive constraint (26) binds: $\Delta_1^* = 0$)

$(b)\,\mu_1 = 0, \mu_2 > 0$ (Only the participation constraint for the loser in the ex post competition binds: $V_{L2}^* = 0$)

$(c)\,\mu_1 > 0, \mu_2 > 0$ (Both ex-ante and ex-post global incentive constraints bind: $\Delta_1^* = 0$ and $V_{L2}^* = 0$)

In all these cases, since $\dfrac{\partial \Delta_1}{\partial \psi} > 0$ and $\dfrac{\partial \Delta_1}{\partial W} > 0$, by the total differentiation of $\Delta_1^*(\psi, W) = 0$, we have the

Marginal Rate of Transformation (MRT) between ψ and W on $\Delta_1^* = 0$, $\left.\dfrac{dW}{d\psi}\right|_{\Delta_1^* = 0} = -\dfrac{\dfrac{\partial \Delta_1}{\partial \psi}}{\dfrac{\partial \Delta_1}{\partial W}} < 0$.

Similarly from the total differentiation of $V_{L2}^* = 0$, we have the Marginal Rate of Transformation between

ψ and W on $V_{L2}^* = 0$, $\left.\dfrac{dW}{d\psi}\right|_{V_{L2}^* = 0} = -\dfrac{\dfrac{\partial V_{L2}^*}{\partial \psi}}{\dfrac{\partial V_{L2}^*}{\partial W}} > 0$, due to the fact that $\dfrac{\partial V_{L2}^*}{\partial \psi} < 0, \dfrac{\partial V_{L2}^*}{\partial W} > 0$.

On the other hand, by the total differentiation of $\pi^*(\psi, W) = \bar{\pi}_p$, we have the principal's the Marginal Rate of

Substitution (MRS) between ψ and W on $\pi^*(\psi, W) = \bar{\pi}_p$ as follows.

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$$MRS_{\psi W}^{P} = \left. \frac{dW}{d\psi} \right|_{\pi^*=\pi_p} = -\frac{\dfrac{\partial \pi^*}{\partial \psi}}{\dfrac{\partial \pi^*}{\partial W}} = -\frac{2(1+t)\left[\dfrac{\partial h^*}{\partial \psi}\right] + \left[(e_W^* - e_L^*) + \dfrac{\partial e_L^*}{\partial \psi} + \psi \cdot \dfrac{\partial (e_W^* - e_L^*)}{\partial \psi}\right]}{2(1+t)\cdot\left[\dfrac{\partial h^*}{\partial W}\right] + \left[\dfrac{\partial e_L^*}{\partial W} + \psi \cdot \dfrac{\partial (e_W^* - e_L^*)}{\partial W}\right] - 1}$$

This implies a balance between the effects of two incentive instruments ψ and W on the sum of ex-ante and ex-post incentives, in other words, the principal's *relative valuation* between ψ and W.

In each of the above three cases, we see that the following corresponding condition should hold at the optimal solution $\left(\psi^*, W^*\right)$.

(a) $MRS_{\psi W}^{P} = MRT_{\psi W}^{\Delta_1^*=0} \Leftrightarrow \dfrac{\dfrac{\partial \pi^*}{\partial \psi}}{\dfrac{\partial \pi^*}{\partial W}} = \dfrac{\dfrac{\partial \Delta_1^*}{\partial \psi}}{\dfrac{\partial \Delta_1^*}{\partial W}}$

(b) $MRS_{\psi W}^{P} = MRT_{\psi W}^{V_{L2}^*=0} \Leftrightarrow \dfrac{\dfrac{\partial \pi^*}{\partial \psi}}{\dfrac{\partial \pi^*}{\partial W}} = \dfrac{\dfrac{\partial V_{L2}^*}{\partial \psi}}{\dfrac{\partial V_{L2}^*}{\partial W}}$

(c) $\dfrac{\dfrac{\partial \pi^*}{\partial \psi}}{\dfrac{\partial \pi^*}{\partial W}} = \dfrac{\mu_1^* \dfrac{\partial \Delta_1^*}{\partial \psi} + \mu_2^* \dfrac{\partial V_{L2}^*}{\partial \psi}}{\mu_1^* \dfrac{\partial \Delta_1^*}{\partial W} + \mu_2^* \dfrac{\partial V_{L2}^*}{\partial W}}$

In case (a), only the first period global incentive constraint is binding. Then the first order condition yields $MRS_{\psi W}^{P} = MRT_{\psi W}^{\Delta_1^*=0}$. Thus we have a standard tangency solution with the first period global incentive constraint. This case may occur because the second period (ex-post) loser's participation constraint is everywhere below the first period global incentive constraint, or because the (indirect) indifference curve of the principal are such that a tangency occurs only on the part of the boundary of the feasible set formed by the first period global incentive constraint. This is a case in which the agents have relation specific capitals enough to have the second period (ex-post) loser's participation constraint non-binding, in other words, the first period capital accumulation is valuable enough.

In case (c), both the first period global incentive constraint and the second period (ex-post) loser's participation constraint become equalities and as a result we can implicitly solve them for the optimal values $\left(\psi^*, W^*\right)$.

Diagrammatically, as figure 5 shows, this solution occurs at the intersection of the two curves corresponding to the two constraints. The lowest possible (indirect) indifference curve touches the kink formed by the intersection of the curves. The feasible set in this case is, of course, the shaded area. We can use the first order conditions to obtain the condition as above, which says that the marginal rate of substitution lies between the values $\dfrac{\partial \Delta_1}{\partial \psi} \Big/ \dfrac{\partial \Delta_1}{\partial W}$ and $\dfrac{\partial V_{L2}^*}{\partial \psi} \Big/ \dfrac{\partial V_{L2}^*}{\partial W}$, as is readily confirmed in the diagram, and at the optimum, a balance relation

holds:

$$\frac{\partial \pi^*}{\partial \psi} = \mu_1^* \frac{\partial \Delta_1}{\partial \psi} + \mu_2^* \frac{\partial V_{L2}^*}{\partial \psi} \quad \text{and} \quad \frac{\partial \pi^*}{\partial W} = \mu_1^* \frac{\partial \Delta_1}{\partial W} + \mu_2^* \frac{\partial V_{L2}^*}{\partial W}.$$

In general, the expression of the Lagrangean is not concave in ψ and W, since the responsiveness of the incentives to ψ and W is involved, in particular, the global incentive constraints may not generally be convex, and the principal's (indirect) indifference curve includes the effects of two incentive instruments ψ and W on the sum of ex-ante and ex-post incentives, and her relative valuation between ψ and W is not simple. So, characterization of the optimal ψ and W is involved, and full characterization of the overall solution would be difficult.[22] The problem for the principal is to maximize the Lagrangean (constrained objective function) subject to the two inequality constraints, noticing the results of table 1. Considering the implications of the first order conditions with respect to e_W^*, e_L^*, h^* and ψ, W, we obtain the following propositions on the partial characterization of the optimal allotment (production share or quantity share ordered by the principal) and the second period prize (monetary payment).

Table 1 around here

[Proposition 8] Inner Solution: Result 1

When the principal decreases ψ (the allotment to the winner in the first period) marginally from 1, if the effect which generates the ex post competition (an increase in the average investment level) is larger than the decrease in the incentive of the first period and the increase in the *virtual cost* which arises in the control of the dynamic competition (the increase in the cost of the global incentive constraint), then $\psi = 1$ (a corner solution viewed as "*Elimination Tournament* (separate the loser)") is not optimal from the viewpoint of the principal.

This is due to the following reasons. If the principal commits herself to $\psi = 1$ at the ex ante stage, the situation of the ex post stage is nothing but a monopoly consisting of the winner, and so, the marginal incentive characterized by $C'(e_s) = \alpha$ arises. On the other hand, a marginal decrease (perturbation) from $\psi = 1$ to $\psi = 1 - \xi$ leads to *a discrete (1-order) loss of the first period incentive* through a discrete change in the second period prize from the monopoly rent to the duopoly rent. But, as for the ex post stage, the decrease from $\psi = 1$

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[22] $\psi < 1/2$ is not optimal for the principal. It is strictly dominated by $\psi = 1/2$.

to $\psi = 1 - \xi$ brings about *a discrete increase in ex post incentives*, which is in the first order. Concretely, evaluating the latter part of the contents of the first brackets {} in the equation (33) at $\psi = 1$, which represents the marginal change in the ex post equilibrium incentives through the marginal change in ψ, we get

$$\alpha + \left[\frac{\partial e_W^*}{\partial \psi} \right]_{\psi=1} = \alpha + \alpha \left((\phi' \cdot W)^2 + (\phi' \cdot W) + 1 \right) > \alpha = C'(e_S) \tag{35}$$

(Here, we assumed the specific cost function $C(e) = (1/2) e^2$ in order to get a clear result. We should investigate more general cost functions, but it is expected that the essence will be the same as above.) We obtain (35) from the process of computation of the comparative statics. Noticing that the second term is strictly positive, if we reduce the value of ψ from 1 to $1 - \xi$, the first order gain, which we shall call the *"competition generating effect"*, emerges with respect to the ex post capital accumulation. Next, $\mu_1 \cdot \left[\frac{\partial \Delta_1}{\partial \psi} \right]$ represents the enhancement of her profit through *relaxation* of the ex ante global incentive constraints in the first period, by increasing ψ. On the other hand, $\mu_2 \cdot \left[\frac{\partial V_{L2}^*}{\partial \psi} \right]$ represents the reduction of her profit through *tightening* of the ex post global incentive constraint, which implies that it tends to be difficult for the loser to reenter the competition, and so, it becomes difficult or virtually costly for the principal to induce the loser to participate in the ex post competition.

The sum of these two terms: $\mu_1 \left[\frac{\partial \Delta_1}{\partial \psi} \right] + \mu_2 \left[\frac{\partial V_{L2}^*}{\partial \psi} \right]$ implies <u>an increase in the incentive costs</u> or an *increase in the cost of managing competition*. These additional terms could be called <u>the total virtual costs</u> [23] of managing competition. This shows how the principal should pay (impose) the reward (penalty), using the allotment ψ as an incentive instrument. Since the negative effect upon the ex ante incentives is discretely large as ψ moves from 1 to $1 - \xi$, when "the competition creating effect in the second period", which dominates over the negative incentive effect in the first period, is larger than the increase in the incentive cost of managing competition

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[23] This concept can be viewed as an application of the concept of 'control cost' in the literature concerning the mechanism design and Bayesian games. Originally, it is a cost required for the principal to induce the revelation of the private information from the agent. For the theoretical literature, see Myerson, R (1981) and chapter 7 of Fudenberg, D and J.Tirole (1991). As an application to the theoretical I.O field, see, for example, Lewis, T and D.Sappington (1991), in which the optimal choice between the subcontracting and the inside production in the firm is investigated, from the viewpoint of the tradeoff between control cost brought about by the private (type) information of the subcontractor and its technological advantage over inside production.

evaluated at $\psi = 1$, the principal should reduce ψ from 1, not very marginally but in some discrete manner[24], establishing the ex post competition for the payment W in order to increase her profit. This means that *nonelimination tournaments*, letting the loser reenter the competition in the 2-period, are preferable, in terms of the total balance of the ex ante and ex post effects, especially in the situations where the second period capital accumulation is relatively vital. This is <u>case (c)</u>, with both ex-ante and ex-post constraints binding.

Proposition 8 tends to be satisfied as the rate of technology transfer: t approaches 1. The reason for this is that the virtual expected profit from the second period competition is increased from the viewpoint of the principal, due to the fact that the equilibrium incentives *ex post* increases as t approaches 1 (Proposition3) and also that the ex post incentive constraint tends to be satisfied/relaxed. In this case, it doesn't contradict the intuition that $\psi = 1$ (the principal gives a monopoly rent to the winner) is not optimal.

Next,

[Proposition 9] Inner Solution: Result 2

As the uncertainty at the end of the first period is smaller, as the bargaining power α of the agents and the rate t of technology transfer are greater, $\psi = 1/2$ is not optimal, in other words, adopting an allotment scheme $\psi > 1/2$ tends to be optimal, or *"endogenously" chosen* by the principal.

The economic intuition with respect to this proposition is the same as that in the case of proposition 8. When the principal commits herself to $\psi = 1/2$, the equilibrium incentive in the second period is characterized by the marginal condition

$$\frac{\alpha}{2} + \phi(0)W = C'(e^*) \tag{36}$$

On the other hand, as for the investment decision in the first period, the two agents simultaneously and independently decide how much to invest, expecting to obtain the reduced form equilibrium expected rent $[2\Phi(K_{i2} - K_{j2}) - 1] \cdot W$, given the difference in capital stocks $K_{i2} - K_{j2}, \ i \neq j$ at the start of the second period subgames. Agent i solves the following problem, given the incentive of agent j, h_j.

$$\underset{\{h_i\}}{Max} \frac{\alpha}{2} \left[\bar{K} + h_i + e^* \right] + F(h_i - h_j) \left[2\Phi(K_{i2} - K_{j2}) - 1 \right] W - g(h_i) \quad i = 1,2, \quad i \neq j$$

The first order condition with respect to h_i is,

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[24] Only case (a) or (c) of the inner solution cases can be optimal. In case (b), where *only* ex post global incentive constraint $V_{L2}^* \geq 0$ is binding, it is better for the principal to *eliminate* the loser, and choose a corner solution $\psi^* = 1$.

$$\frac{\alpha}{2} + f\left(h_i - h_j\right)\left[2\Phi\left(K_{i2} - K_{j2}\right) - 1\right]W + F\left(h_i - h_j\right)\left[2(1-t)\phi\left(K_{i2} - K_{j2}\right)\right]W = g'\left(h_i\right) \quad i = 1, 2, \ i \neq j$$

The first term is the share α of the marginal increase of the quality, under the expected allotment of $1/2$. The second term is the increase of the expected rent in the second period through marginal improvement of the probability of winning in the first period, and the third term is the marginal increase in the expected rent in the second period through marginal improvement of the probability of winning in the second period. Now, we restrict our restriction to the symmetric equilibrium in the first period. Then, the second term disappears in equilibrium, and we obtain the following equation, characterizing the equilibrium investment level.

$$\frac{\alpha}{2} + (1-t)\phi(0)W = g'\left(h^*\right) \tag{37}$$

If the agents play the "two stage" investment game under the regime of $\{\psi = 1/2 \text{ and } W > 0\}$ the equilibrium investment levels are characterized by both (36) and (37). In the left hand side of both equations, the first term corresponds to *the direct effect* such that the agents expect to get the allotment $1/2$ in equilibrium and receive the share α of one unit increase in capital accumulation through their investments, as private revenue. The second term is called *the (marginal) strategic effect,* which is the product of the monetary payment and the marginal improvement of the probability of obtaining W. The sum of these two corresponds to the marginal benefit of the investment in each period. Note that in (37), if $t = 1$, the strategic effect does disappear, because the agent's performance (accumulation) is perfectly transferred to the rival agent, so that he/she is perfectly exploited by the rival the progress attained through his/her costly effort. Next, we consider a marginal change in ψ *from* $1/2$ *to* $1/2 + \xi$. Then, the incentive loss in the second period is *only in the second order* through the marginal change in the equilibrium level. However, as for the first period, the discrete prize can be obtained when the agent's own performance is superior to the rival's performance, in other words, *a discontinuous jump* arises in the reward function, at the level of the rival's capital accumulation achieved. This will generate a discrete increase in the first period incentives. (See Figure 3)

The essential point can be stated as follows. When $\psi = 1/2$, the marginal strategic effect arises through the observability of the progress along the way at the end of the first period. On the other hand, when $\psi = 1/2 + \xi$, where ξ is small, *the discrete tournament effect* emerges through the discrete prize (16), which increases the first period equilibrium incentives discretely. Hence, the principal can induce more incentives in terms of intertemporal incentives by perturbing the allotment from $\psi = 1/2$ to $1/2 + \xi$. Technically, the necessary and

sufficient condition is that $\left|MRS_{\psi W}^{p}\right| < \left|MRT_{\psi W}^{p}\right|_{\Delta_1=0}$ evaluated at $\psi = 1/2$ [25]. This could be simplified using the

result so far, but it is easily checked that as the uncertainty is smaller, i.e, $f(0)\uparrow \Rightarrow h^{*}\uparrow$, and as the bargaining

power α of the agents and the rate t of technology transfer are greater, this inequality tends to hold. Also, it is

sufficient if the following condition

$$\xi \geq \frac{1}{[\alpha]\cdot[f(0)]} \tag{38}$$

is satisfied [26]. This is a sufficient condition of the optimality of adopting a two stage tournament (the allotment

and the monetary prize). (38) tends to be satisfied, as f (0) is relatively large, in other words, the uncertainty is

relatively small. Let us mention the above argument from the viewpoint of *the incentive cost*. We can say that

the principal can increase the equilibrium profit, by adopting the allotment scheme, because he can reduce the

incentive cost (the implementation cost, in this case, the monetary reward or premium) to induce a constant level

of incentives on the equilibrium path.

3.3.3.2.Corner Solution Cases

In this corner solution cases, there are the following two cases[27].

(d) $\psi^{*} = 1/2 \quad (W^{*} > 0)$

$(e)\ \psi^{*} = 1 \quad (W^{*} > 0)$

Case (d) should be companioned with Proposition 9 and its analysis. In this case, the relation that

$\left|MRS_{\psi W}^{p}\right| \geq \left|MRT_{\psi W}^{p}\right|_{\Delta_1=0}$ should hold, evaluated at $\psi^{*} = 1/2$. This tends to hold as $\alpha \to 0$. This result is

consistent with the comparative statics conjecture that as $\alpha \to 0$, the principal should concentrate the

procurement on the winner at the end of the second period. See Discussion 4.2.

Case (e) should be companioned with case (b) and Proposition 8. At $\psi^{*} = 1$, the constraint $V_{L2}^{*} \geq 0$

automatically holds with $W > 0$. So, we should eliminate the case that $V_{L2}^{*} > 0$. Then, when perturbing the

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[25] That is, evaluated at $\psi = 1/2$, $\dfrac{2(1+t)\left[\dfrac{\partial h^{*}}{\partial \psi}\right] + \left[(e_{w}^{*}-e_{L}^{*}) + \dfrac{\partial e_{L}^{*}}{\partial \psi} + \psi \dfrac{\partial(e_{w}^{*}-e_{L}^{*})}{\partial \psi}\right]}{2(1+t)\left[\dfrac{\partial h^{*}}{\partial W}\right] + \left[\dfrac{\partial e_{L}^{*}}{\partial W} + \psi \dfrac{\partial(e_{w}^{*}-e_{L}^{*})}{\partial W}\right] - 1} < \dfrac{\alpha\left\{(\bar{K}+h_{L}+th^{*}) + 1/2(e_{w}^{*}+e_{L}^{*})\right\}}{\Phi^{*}-1/2}$

[26] Readers can request the details of the derivation of this sufficient condition from the author. He is investigating other sufficient

conditions and a method, which may depict the structure of the overall game more clearly.

[27] We assume inner solutions $W^{*} > 0$.

incentive scheme ψ from $\psi = 1-\xi$ to $\psi = 1$, a *discrete reduction of Incentive Cost* is brought about at $\psi^* = 1$. This <u>discontinuous jump</u> of incentive constraint $V_{L2}^* \geq 0$ at $\psi = 1$ may make it better for the principal to choose the corner solution $\psi^* = 1$ than an inner solution case (b) with only $V_{L2}^* \geq 0$ binding, because it improves the principal's profit greatly *through a discrete reduction of incentive cost*.[28] Locally, the condition that $\left| \dfrac{\partial \pi^*}{\partial \psi} \Big/ \dfrac{\partial \pi^*}{\partial W} \right| < \left| \dfrac{\partial \Delta_1}{\partial \psi} \Big/ \dfrac{\partial \Delta_1}{\partial W} \right| \Leftrightarrow \left| \mathrm{MRS}_{\psi W}^P \right| < \left| \mathrm{MRT}_{\psi W}^{\Delta_1=0} \right|$ holds at the corner $\psi = 1$ on $\Delta_1(\psi, W) = 0$. This implies that the principal would choose $\psi^* = 1$ as a local optimum, just as the necessary and sufficient condition for the principal to choose $\psi^* > 1/2$ is $\left| MRS_{\psi W}^p \right| < \left| MRT_{\psi W}^p \big|_{\Delta_1=0} \right|$ evaluated at $\psi = 1/2$.

Last, we obtain the following proposition about the monetary prize (efficiency wage) W.

[Proposition 10]

It is optimal to set the positive monetary prize W in the final period in the case of a non-elimination tournaments $\left(\text{i.e. } \psi > 1/2, W > 0 \right)$

As we verified before, $\dfrac{\partial \Delta V}{\partial W} = 2\Phi(\Delta K + e_W^* - e_L^*) - 1 > 0$. Hence, the intuition with respect to this proposition is clear. If the principal increases W under the condition that $\dfrac{\partial \Delta V}{\partial W} > 0$, this leads not only to an increase in the investment levels $\left(e_W^*, e_L^* \right)$ in the second period, but also an increase in the equilibrium investment level h^* in the first period through an increase in size of the prize ΔV, as the payoff difference generated in the asymmetric equilibrium in the second period. Hence, the prize W set for the final winner has *a long term effect*, and it is a device, combined with the allotment scheme, for inducing incentives from agents over the two periods.

4. Discussions and Some Extensions

4.1 Comparative Statics: the effects of exogenous parameters upon the set of credible contracts and optimal

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[28]As for a similar idea, see Itoh (1991). In a *static* multiple agents contracting model, he studies whether the principal should optimally induce a helping effort from each of multiple agents. Net benefit vs. *incentive cost* from inducing a help determines the solution on this problem. In cases where a *discrete jump of incentive cost* occurs when *marginally* inducing a helping effort, the principal shouldn't induce any help from the agents. This is just a corner solution like ours.

incentive schemes

(1) <u>The bargaining power α of the agents</u>: a 'zero-sum' factor between the principal and the agents.

This parameter reflects how the degree of *technological initiative* in development activities of each part (component) is assigned between an assembler and a part supplier. As α is larger, the component tends to be one of DA ("drawing approved") parts, since DA suppliers are providing not only capabilities for *manufacturing* of parts transacted, but also capabilities for *product development*, and so the degree of their technological initiative and their bargaining power is higher. While, as α is smaller, the component tends to be one of DS ("drawings supplied") parts, since DS suppliers are providing basically only capabilities for *manufacturing* of the parts transacted, and so the degree of their technological initiative and their bargaining power is lower.

As α becomes bigger, the participation constraints for the agents become more relaxed. Hence, the principal can reduce W for each ψ, in order to induce the agents to participate in the competition, with the first period global incentive constraints satisfied. The effect on the second period participation constraint for the loser $V_{2L}^* \geq 0$ is a bit complex. As proposition 5 shows, when α increases, for $\psi \in [\overline{\psi}, 1)$, where $\overline{\psi} \in (1/2, 1)$ is a critical level, it has the best response function of the winner shift outward even more than the loser's one. It is due to a big difference in marginal productivities $\alpha\psi - \alpha(1-\psi) = \alpha(2\psi - 1)$. Then, the loser's incentive e_L^* greatly decreases along his best response function, due to the increase of the winner's investment, since the investments are *strategic substitutes* for the loser. We call this "the disincentive effect", which has already been explained as a "*feedback effect*". On the other hand, due to the increase of the bargaining power α, the 'share' of the revenue per unit for the loser increases. We call this "the distribution effect", which has already been explained as a "direct effect". When ψ is relatively greater, "the disincentive effect (feedback effect)" dominates "the distribution effect（direct effect）". Hence, $\partial e_W^* / \partial\alpha > 0$ and $\partial e_L^* / \partial\alpha < 0$ holds in equilibrium. This tightens $V_{2L}^* \geq 0$, and so the principal must increase W for each ψ, in order to increase the fascination of the competition. However, for $\psi \in (0, \overline{\psi})$, where $\overline{\psi} \in (1/2, 1)$, "the distribution effect (direct effect)" dominates "the disincentive effect (feedback effect)". Hence, an increase of α brings about $\partial e_L^* / \partial\alpha > 0$, which relaxes $V_{2L}^* \geq 0$. From these investigation, we see that the loser's participation constraint $V_{2L}^*(\psi, W) \geq 0$ shift downs for $\psi \in (0, \overline{\psi})$, but shifts up for $\psi \in [\overline{\psi}, 1)$. When $\overline{\psi}$ is smaller than ψ^*, the set of credible contracts expands upward and downward around the left hand side of the original optimal solution, due to the downward shifting of both global incentive constraints $\Delta_1^* \geq 0$ and $V_{2L}^* \geq 0$. Thus, the new optimal levels of both ψ^* and W^* would become smaller than before. This implies that α as an incentive device *complements* the function as an incentive device of (ψ, W) in the inner solution. In other words, we could also say that because the loser's participation constraint $V_{2L}^*(\psi, W) \geq 0$ shifts up for $\psi \in [\overline{\psi}, 1)$, the corner solution $\psi^* = 1$ (elimination tournaments) tends to be optimally chosen, due to the increase of incentive costs for inducing the loser to

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participate in the ex post competition. Last, when $\alpha \to 0 \Leftrightarrow 1-\alpha \to 1$, if the principal chose <u>one-stage</u> <u>tournaments</u> with $\psi \to 1$, the induced incentives on the equilibrium path would lead to the first best. As an example, we consider <u>the Drawings Supplied Parts</u>, such as subassembly[29].

(2) <u>The demand (total trade volume)</u>: If we model that the demand for the final goods is Q instead of 1, it is a 'non-zero- sum' factor between the principal and the agents. An interesting case is the case where Q decreases. As it decreases, the incentive levels by themselves decrease. But, from the F.O.Cs (22), the competitive pressure for the allotment tends to push the agents' incentives over the hold up levels h_s. This suggests that this kind of procurement mechanism with supplier competition tends to be more favorable than the one with a single supplier, when the market is *in a slump* .In practice, the Japanese subcontracting system with supplier competition ('Managed competition ') was established after the experience of the oil crisis.

(3) <u>The technology transfer rate</u> t: the degree of information transfer/sharing

Let us investigate how t affects the set of credible contracts, especially, the global incentive constraint of the loser in the second period. Needless to say, as t increases, not only will the ex post equilibrium incentives increase, but also the global incentive constraint for the loser will be relaxed. Hence, the expected profit to be obtained by the principal will increase from the viewpoint of the second period. Next, we shall investigate the effect upon the first period. Noticing that the technology transfer is made *after* the interim ranking with the allotment, the technology (information) transfer will increase the first period prize, as is evident from (16), and so the first period equilibrium incentives. Thus, the equilibrium profit to be obtained by the principal in the first period will also increase.

4.2 Incentive constraints for the principal for ψ *and W*: *Dynamic Self-Enforcement*

The relative ranks of the levels of capital are *unverifiable* .If so, we will have to consider the principal's incentive for enforcing the allotment scheme ψ and the wage payment W . This is a standard *self-enforcing constraint* imposed on the principal in the contexts of repeated relationships.[30] As is well known in the repeated

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[29] See Asanuma (1989) for the classification of the parts transaction relationship between manufacturer and suppliers.

[30] Recent 'relational contracts' literature analyzes this point explicitly. For example, Baker, Gibbons and Murphy (2002) examines in an integrative form an interaction between asset ownership (*who owns the asset*) and relational contracts-contracts that are self-enforced through repeated interaction, both within (*employment*) and between (*outsourcing*) firms. Our framework belongs to '*relational outsourcing*' between firms (the upstream suppliers also own the asset e.g. product machinery) in their classification, though we don't explicitly present the dynamic self-enforcing constraint in the model. While, they don't analyze the effect of dynamic competition

game literature, if the discount factor of the principal is sufficiently high, the optimum solution $\{\psi^*, W^*\}$ by the principal (buyer) in section 3 will satisfy her dynamic self-enforcement constraint. But, if the discount factor is low, her dynamic self-enforcement constraint will not be satisfied at $\{\psi^*, W^*\}$. This increases the number of constraints that the initial contract $\{\psi, W\}$ must satisfy, and so, will reduce the attainable equilibrium profit to be obtained by the principal.

4.3 The relation specificity and the effect of the outside options on the sustainability of 'Managed Competition'.

The intuition is that as the relation specificity is reduced, the outside options increase and it becomes harder to induce the loser to participate in the ex post competition in organizations, so the principal must compensate the loser for participation by incurring the cost of more wage payment. Since the late 1990s, it tends to be more difficult to maintain managed competition than before, since the value of outside options has become higher, which tightens the participation constraint of the first period loser in the ex post competition, and so it becomes more costly for the principal to provide incentives for the loser to participate also in the ex post competition. This is represented by the inequality observed as the RHS of the loser's ex post participation constraint: $V_{L2}^* \geq 0$ increases above zero, so that it tends not to be satisfied. This theoretical intuition may correspond to the practice observed in the case of small Japanese companies (suppliers), which are trying to expand business overseas through the production of parts and trade.

4.4 Partially contingent vs. fully contingent allotment scheme:

In the main text, we assumed the following form of allotment scheme.
$$\Psi_i\left(\widetilde{K}_{i1}, \widetilde{K}_{j1}\right) = \begin{cases} \psi & if \ \widetilde{K}_{i1} > \widetilde{K}_{j1} \\ 1-\psi & otherwise \end{cases}$$. This allotment scheme is a partially contingent allotment scheme, which

depends only on the realization of the interim rank and <u>a constant allotment</u> ψ or $1-\psi$ over each state ($\widetilde{K}_{i1} > \widetilde{K}_{j1} \Leftrightarrow \omega_i$ or $\widetilde{K}_{i1} < \widetilde{K}_{j1} \Leftrightarrow \omega_j$) is assigned.

If we suppose that a *more flexible* allotment scheme exists such that the principal changes the allotment share, depending on the difference in the end of first period capital stocks $\widetilde{K}_{i1} - \widetilde{K}_{j1}$, as well as the rank information ω, that is, $\Psi_i\left(\omega, \widetilde{K}_{i1} - \widetilde{K}_{j1}\right)$ it may increase the equilibrium payoff obtained by the principal, because the incentive scheme becomes more flexible in terms of the information regarding the states. We may justify the

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between the two upstream suppliers and its management by the downstream firm (the principal) in procurement.

assumption of constant allotment as follows. First, the difference of capital stocks $\tilde{K}_{i1} - \tilde{K}_{j1}$ may be observable, but very hard to explain, give proof (even soft evidence), and it may be difficult to persuade (convince) them. This can be interpreted as a kind of verification (monitoring) cost. Let us define the verification cost as $H\left(\left|\Delta\tilde{K}_{i1}\right|\right)$, with $H' > 0$, $\forall\left|\Delta\tilde{K}_{i1}\right| \geq 0$, where $\left|\Delta\tilde{K}_{i1}\right| = \left|\tilde{K}_{i1} - \tilde{K}_{j1}\right|$. Especially, it will be important that the marginal verification cost at zero $H'(0)$ is positive and large enough. In such cases, it may be better for the principal to choose a partially contingent, constant allotment scheme, which depends *only on the ranking*, in other words, the realization of ω_i or ω_j, as discussed above, rather than choosing a more flexible (contingent) allotment scheme.[31]

4.5 Open Loop Strategy vs. Closed Loop Strategy in Managed Competition

Depending on the information structure assumed for the agents, several possibilities as to the choice of the control variables are available[32]. Let us investigate two major strategy spaces; open loop vs. closed loop strategies. Here, we deal only with the case of perfect state information. In the open loop strategies, the agents commit themselves to specific time functions, and will not adjust the strategy in response to changes in the state of the game of the second stage. Hence, open loop strategies are called *path strategies*, and are only a function of time, given the initial time: $\overline{u}$ at the start of the second period. Here, the Nash Equilibrium in the second period $\left(e_W^*, e_L^*\right)$ can be viewed as an open loop Nash equilibrium, where each agent's (especially, the loser's) path is the best response to its rival's (the winner's) path. Only under such information structure, the loser will be able to gain a come-from-behind victory in the second period, and obtain a lumpy prize: W. Thus, the principal can induce effective competition also in the second period.

On the other hand, in the closed loop strategies, the strategy space of each player is a set of decision rules that depend on time: u and the current state .The agent i does not precommit to a particular path of control variables, but rather responds to different current states he observes. In equilibrium, closed loop strategies of agent i always remain an equilibrium strategy for any possible subgame of the game. Thus, the agents select mutual best responses for each other at all feasible points; $\forall\left(\left(K_{W2}(u), K_{L2}(u)\right), u\right)$ for $\overline{u} \leq u \leq T$ and for every initial condition (at $\overline{u}$); (ψ, W). So, they are subgame perfect and time consistent in the sense of Selten (1975).

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[31] This idea may be close to Bernheim and Whinston (1998), which argues that when some performance measures ($\tilde{K}_{i1} - \tilde{K}_{j1}$) are unverifiable, parties may deliberately leave formal contracts ($\Psi_i\left(\tilde{K}_{i1}, \tilde{K}_{j1}\right)$) *incomplete*, that is, "strategically ambiguous". As an interpretation, the contract form in our paper satisfies the "strategic ambiguity" described by them, since principal specifies a constant allotment $(\psi, 1-\psi)$, but leaves the possibility of a come-from-behind victory and giving a final reward *ex post*, that is, *flexibility* in the sense that the victory is not fixed only in the first stage.

[32] For detailed discussion, see Petit (1990)

In the case of such information structure, through the logic of subgame perfection, only the winner in the first period will continue to invest in the Nash equilibrium in closed loop strategies. Hence, if the close loop strategies are available among agents, the principal should choose the elimination tournament $\left(\psi^* = 1, W > 0\right)$. In this case, the non-elimination tournament $(1/2 <)\psi^* < 1$ is dominated by the elimination tournament. However, we can state that when the principal selects a non-elimination tournament, the principal, as a necessary condition, should adopt *open loop* strategies among the agents, or control the information structure such that only the open loop strategies are available, in other words, the agents cannot observe the evolution of state (the capital accumulation process in the second period competition) among them.

5. Concluding Remarks

In this paper, we modeled a supply relation where a fixed, few part suppliers compete dynamically for a position of supply source of parts and we investigated how a limited, but active management policy of competition by the principal affected the dynamic incentives of the agents and could induce the higher capital accumulation. In the Japanese style competition, it is often said that there does not exist a loser in the true sense, since these are "non-elimination" tournaments. When the principal cannot explicitly write and commit herself to a state-contingent initial contract, the 'Hold- up problem' often occurs, leading to underinvestment problems[33]. Even in such situations, the principal can design multi-dimensional races, and provide the first period incentives, with the agents mainly seeking for a "non-monetary" reward (larger allotment), and at the same time, maintaining the interim equality in terms of capital accumulation levels through technology transfer, and induce the active competition for another prize even in the second period. This type of competition could be interpreted as the *voice mechanism* suggested by A.Hirshman (1970), especially, the voice mechanism designed by the principal, with monetary and nonmonetary incentive schemes.

Next, in the solutions where $\psi = 1/2$, under the information structure where the relative performance at the interim stage is *observable* by the agents, the subgame played by them is essentially a two-stage game .In this case, as the second term of (37) shows, the ex- ante incentives among the agents are voluntarily increased through the marginal strategic effect. On the other hand, under the information structure corresponding to the *unobservability* of capital stocks at the interim stage in the solutions where $\psi = 1/2$, the two agents will play a

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[33] Even in the world of incomplete contracts, by inherent multiplicity of equilibria, information revelation and incentive compatible *equilibrium selection*, a tournament can be *endogenously* generated to strengthen the incentive for investment in a relation specific skill. As for this, see Konishi, Okuno-Fujiwara and Suzuki (1996).

one-stage game, where they *commit themselves to the capital accumulation path over the two periods,* and so the strategic effect: $(1-t)\phi(0)W$ disappears. This can be stated such that if the principal can manage the

information structure such that the two agents can observe each other's capital accumulation outcome in the first period, more incentives could be induced by $(1-t)\phi(0)W$ in the first period, for a constant prize: W. In

addition, we investigated how the principal strategically utilized the incentive creation effect through a change in information structure, by adopting the allotment scheme. As a result, we showed that under some sufficient conditions, the principal should adopt the allotment scheme $\psi > 1/2$ and could provide a larger incentive through

a discrete prize as a difference in equilibrium payoffs, and also so for the intertemporal incentives. This type of

information management policy and the principal's strategy of utilizing the change in induced dynamic incentives

are essential both theoretically and as an explanation for managed competition.[34]

Let us overview a characterization of the solution to the overall Kuhn-Tucker problem. Under some exogenous conditions, a corner solution where $\psi^* = 1/2$ (not using the two-stage tournaments) or where

$\psi^* = 1$ (elimination of the loser from the ex-post competition) may be chosen as an optimal solution to the

Kuhn-Tucker problem. However, we find that when the market demand is large, the uncertainty is small, the

bargaining power of suppliers α is greater, that is, the parts tend to belong to the *drawings approved* parts, and

the spillover of technology and knowledge tends to occur, then the two-stage tournaments with allotment

$(1/2 < \psi < 1$ and $W > 0)$ tend to be optimally chosen. This condition is commensurate with the situation in the

catch-up economy or the Japanese subcontracting system in the second half of 1980's.

When the exogenous conditions change, how is the optimal solution (the combination of the two strategies:

ψ and W) affected in inner solution cases?   We have obtained the interesting comparative statics on the

bargaining powers of suppliers α and the degree of technology and information transfer t, and the

accompanying result that when the bargaining power of the agent tends to zero, if the principal chose <u>one-stage tournaments </u>with elimination i.e.$\psi \rightarrow 1$, the induced incentives on the equilibrium path would lead to the first

best. As an example, we could imagine the *Drawings Supplied* Parts, such as subassembly.

By the way, "Managed Competition", whose essential aspects are *two supply policy* and *design in*,
is a *self-enforcing institution* a la Greif (2001), in the sense that given the *exogenous parameters* $\left((\alpha, t)\right)$,

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[34] It is well known that a change in information structures brings about a change in strategy spaces, leading to a change in equilibrium

incentives .As for this, see Fudenberg -Tirole (1991). From a practical point of view, Itami (1988) points out how information

management affects the incentives for quality improvement in the Japanese subcontracting system.

discount factor one, no/fixed outside option etc), the strategies of each player constitute a self-enforcing agreement, i.e. subgame perfect Nash equilibrium. Nonetheless, such exogenous parameters might not be exogenous, as time and the "Managed Competition" itself evolve. Then, some part of "Managed Competition" might not be self-enforcing, and so might change. Such dynamic perspective on institutions seems to be interesting and indeed can be applied to the topics of this paper. This would be a possibility for future research.

REFERENCES

Asanuma, B, (1989). "Manufacturer-Supplier relationship in Japan and the concept of relation specific skill", *Journal of the Japanese and International Economies*, 3, 1-30.

Baker, G., R.Gibbons. and K. Murphy. (2002) "Relational Contracts and the Theory of the Firm." *Quarterly Journal of Economics*, February 39-84

Benoit, J.P, and V.Krishna. (1985), "Finitely Repeated Games", *Econometrica*, 53; 890-904.

Bernheim, D and M.Whinston (1998) "Incomplete Contracts and Strategic Ambiguity", *American Economic Review*, 88, 902-932.

Bolton, P and M.Whinston (1993) "Incomplete Contracts, Vertical Integration, and Supply Assurance" *Review of Economic Studies*, 60,121-148.

Clark, K and T.Fujimoto (1991) *Product Development Performance*. Harvard Business School Press.

Fudenberg, D and J.Tirole. (1991) *Game Theory*, MIT Press, Cambridge

Garcia, C.M. (1993) "Vertical Market Structure: A Dual Sourcing Approach," *mimeographed.*

Greif, A (2001) "Institutions in Historical Perspectives" Lecture and Lecture Handouts at Stanford University.

Grossman, S, and O.Hart (1986), "The Costs and the Benefits of Ownership: A theory of Vertical and Lateral Integration", *Journal of Political Economy*, 94, 691-719.

Hart, O. and J.Moore (1990) "Property rights and the nature of the firm", *Journal of Political Economy*, 98, 1119-1158.

Helper, S. (1991) "How much has really changed between U.S. automakers and their suppliers?" *Sloan Management Review*, 15.

Helper, S. (1994) "Supplier Relations in the Auto Industry: A limited Japanese –U.S Convergence? IMVP paper.

Hirshman, A.O. (1970), Exit, Voice and Loyalty, Cambridge Mass: Harvard University Press.

Holmstrom, B and Roberts, J (1998) "The Boundaries of the Firm Revisited", *Journal of Economic Perspectives*,

        Vol.12, No 4, Fall, 73-94.

Itami, H. (1988) Mieru-te-niyoru-kyousou: buhin-kyoukyu-taiseino-kouritusei" (Competition by Visible Hand: Efficiency of Parts Supply Systems), in "Competition and Innovation: Firm Growth in Automobile Industry" (Itami.H, Kagono.H, Kobayashi.T, Sakakibara and Ito.M eds)

Itoh, H. (1991) "Incentives to Help in Multi Agent Situations". *Econometrica*, 59.3:611-36.

Innes, R. (1990), "Limited liability and Incentive Contracting with Ex Ante Action Choices", *Journal of Economic Theory*, 52, 45-67.

Konishi, H., M.Okuno-Fujiwara. and Y.Suzuki. (1996), "Competition through Endogenized Tournaments: An Interpretation of 'Face-to-Face' competition". *Journal of the Japanese and International Economies*. 10, 199-232.

Lewis, T. and D.Sappington (1991), "Technological change and the boundary of the firm" *American Economic Review*, September.

Matsui, A (1989), "Consumer-Benefited Cartels under strategic capital investment competition", *International Journal of Industrial Organization*, 7, 451-470.

Milgrom, P and Roberts, J (1990) "Rationalizability, Learning and Equilibria in Games with Strategic Complementarities" *Econometrica* 58 1255-1278

Milgrom, P., and J.Roberts 1992 *Economics, Organizations and Management*, Prentice-Hall, Englewood Cliffs.

Myerson, R. (1981). "Optimal Auction Design", *Mathmatics of Operations Research*, 6, 58-73.

Nalebuff, B. and J.Stiglitz. (1983) "Prizes and Incentives: towards a general theory of Compensation and Competition", *Bell Journal of Economics*, 2, 21-43.

Petit, Maria Luisa (1990) *Control Theory and Dynamic Games in Economic Policy Analysis*, Cambridge University Press.

Riordan, M (1990) "Asset Specificity and Backward Integration", *Journal of Institutional and Theoretical Economics*.146, 133-146.

Riordan, M (1991) "Ownership without Control: Toward a Theory of Backward Integration" *Journal of the Japanese and International Economies*. 5, 101-119.

Riordan, M and D.Sappington (1989) "Second Sourcing" *Rand Journal of Economics,* 20, 41-58.

Rosen, S. (1986), "Prizes and Incentives in Elimination Tournaments", *American Economic Review*, September 701-715.

Shapiro, C. and J.Stiglitz. (1984) "Equilibrium unemployment as a worker discipline device", *American*

Economic Review, 74, 433-444,

Suzuki, Y. (1998) "Capital Accumulation through 'Controlled Competition': Two Stage Tournaments as a Strategy for Firm Growth". Chapter3 in his PhD dissertation, The University of Tokyo, February 1999.

Tirole, J (1988) Theory of Industrial Organization, Cambridge, MA: The MIT Press.

Williamson, O.E. (1979), "The Transaction cost Economics: The governance of contractual relations". *Journal of Law and Economics*. 22. 2. 233-261
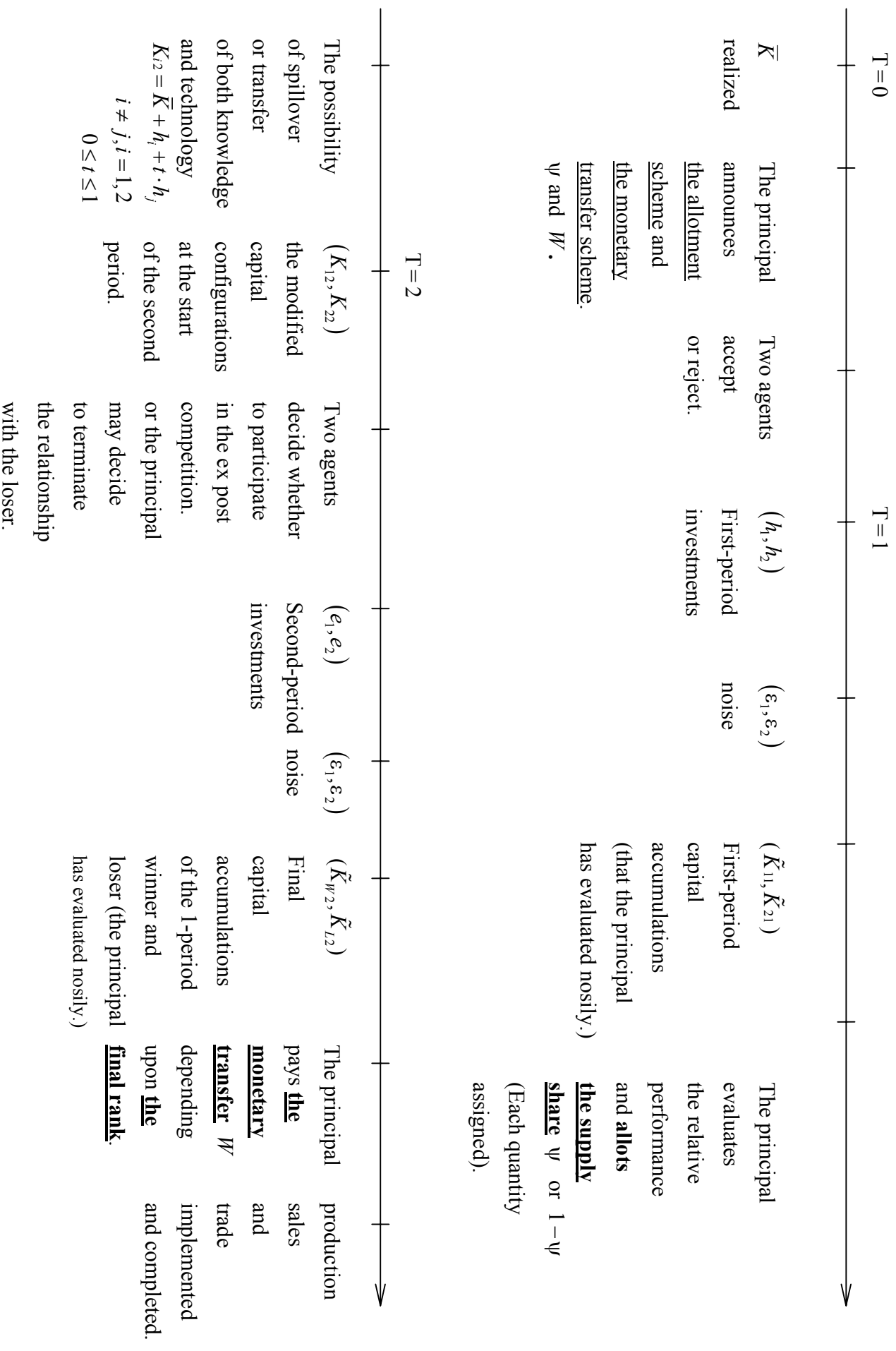
T=0   T=1

$\bar{K}$ realized

The principal announces the allotment scheme and the monetary transfer scheme. ψ and W.

Two agents accept or reject.

(h_1, h_2) First-period investments

$(\varepsilon_1, \varepsilon_2)$ noise

$(\tilde{\bar{K}}_{11}, \tilde{K}_{21})$ First-period capital accumulations (that the principal has evaluated nosily.)

The principal evaluates the relative performance and **allots** **the supply** **share** ψ or 1−ψ (Each quantity assigned).

T=2

The possibility of spillover or transfer of both knowledge and technology
$K_{i2} = \bar{K} + h_i + t \cdot h_j$
$i \neq j, i = 1,2$
$0 \leq t \leq 1$

(K_{12}, K_{22}) the modified capital configurations at the start of the second period.

Two agents decide whether to participate in the ex post competition. or the principal may decide to terminate the relationship with the loser.

(e_1, e_2) Second-period investments

$(\varepsilon_1, \varepsilon_2)$ noise

$(\tilde{K}_{W2}, \tilde{K}_{L2})$ Final capital accumulations of the 1-period winner and loser (the principal has evaluated nosily.)

The principal pays **the** **monetary** **transfer** W depending upon **the** **final rank**.

The principal production sales and trade implemented and completed.

Figure 1

Figure 2.1

Reaction function of the Loser

$\frac{1}{2} < \psi < 1$

$W > 0$

Reaction function of the Winner

e_L

e_L^*

0

e_W^*

e_W

Figure 2.2

The effect of the increase in W

$\frac{\partial \left(e_W^* - e_L^* \right)}{\partial W} \leq 0$

$\frac{\partial e_L^*}{\partial W} > 0$

e_L

Δe_L^*

0

Δe_W^*

e_W

Figure 2.3

The effect of the increase in ψ

$\frac{\partial \left(e_W^* - e_L^* \right)}{\partial \psi} \geq 0$

$\frac{\partial e_L^*}{\partial \psi} < 0$

e_L

Δe_L^*

0

Δe_W^*

e_W
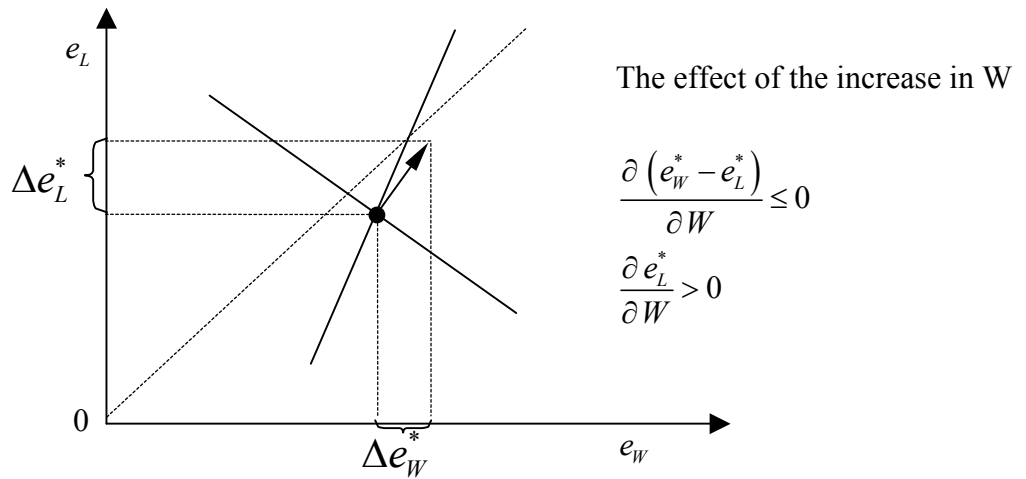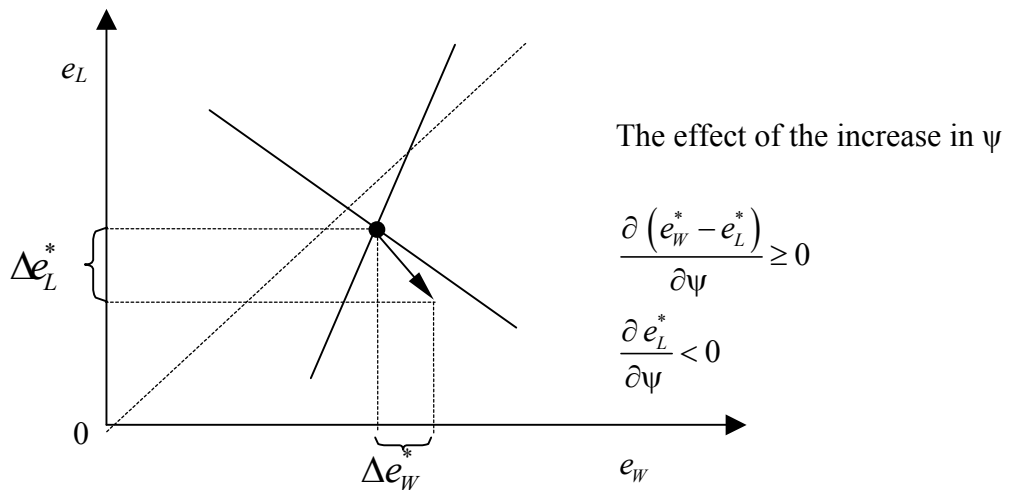
Figure 2·1～2·3   Tue Nash equilibrium in the second period
and the effect of the increase in ψ and W
upon the equilibrium.

The case where
$\frac{1}{2} < \psi < 1$, W>0
a n d $t=0$
(No spillover case)

$S_i(\widetilde{K}_{i1}, \widetilde{K}_{j1})$

$\alpha\psi$

$\Delta V(\psi, W)$

$\alpha(1-\psi)$

0

$\widetilde{K}_{j1}$
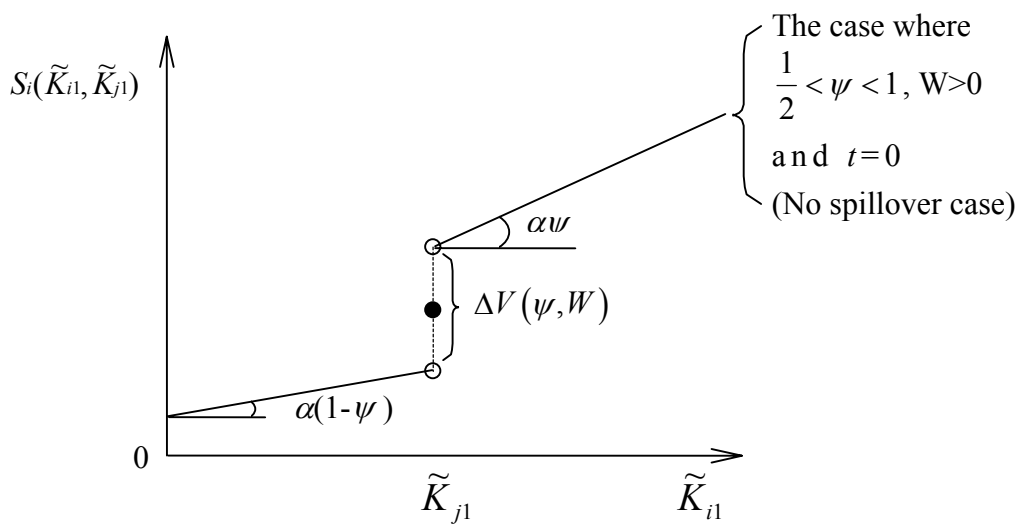
$\widetilde{K}_{i1}$

Figure 3   The Tournament Scheme with which the agent i is
              faced in the first period (This scheme is "endogenously"
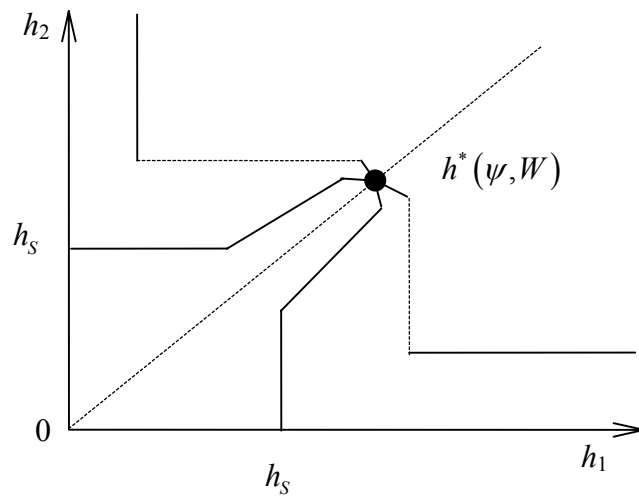   chosen by the principal through the allotment scheme)

h_2

$h^*(\psi, W)$

h_S

0

h_S

h_1

Figure 4   The equilibrium investment level in the first period $h^*(\psi, W)$

Two incentive devices / Benefit and cost	ψ	W
The effect upon the ex ante incentives: $h^*(\psi,W)$	+	+
The effect upon the weighted average of the ex post incentives: $\psi e_W^* + (1-\psi)e_L^*$	- The fall of the "competitive pressure" **Figure 2.3**	+ The revival of the "competitive pressure" **Figure 2.2**
Direct cost for the principal	0	1
The effect upon the ex ante global incentive constraint	+ **(relaxed)**	+ **(relaxed)**
The effect upon the ex post participation constraint of the loser	- **(tightened)**	+ **(relaxed)**

Table 1

The effect upon both the ex ante and ex post equilibrium incentives brought about by the marginal increases of the two incentive devices, in the range of $\frac{1}{2} \leq \psi < 1$ and W≥0).